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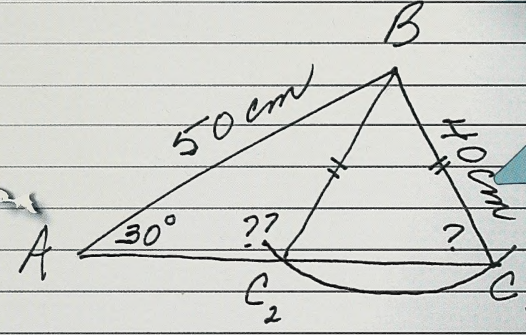
Module 3

MATHEMATICS 20

Quadratics



$$\sin C = \frac{c \sin A}{a}$$

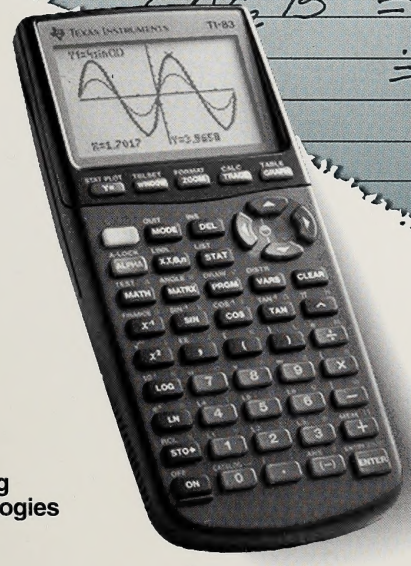
$$= \frac{50 \sin 30}{40}$$

$$\angle C \doteq 38.7°$$

$$\therefore \angle C_1 \doteq 38.7°$$

$$\angle BC_2B = 180 - \angle C_1$$

$$\doteq 141.3°$$



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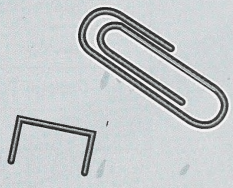
Pure MATHEMATICS 20

Module 3
Quadratics



Learning Technologies Branch

Alberta LEARNING

Pure Mathematics 20
Module 3: Quadratics
Student Module Booklet
Learning Technologies Branch
ISBN 0-7741-1912-8

This document is intended for	
Students	✓
Teachers	✓
Administrators	
Parents	
General Public	
Other	

The Learning Technologies Branch has an Internet site that you may find useful. The address is as follows:

http://www.learning.gov.ab.ca/ltb

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Welcome

Welcome to Module 3. We hope you'll enjoy your study of Quadratics.

Pure Mathematics 20

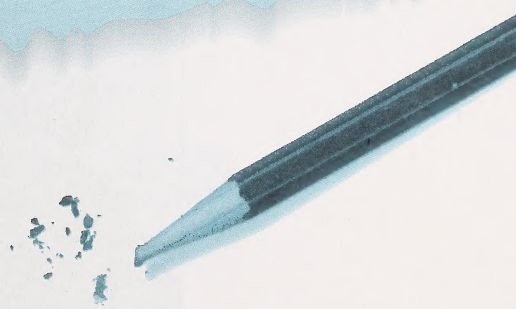
Module 1
Consumer Mathematics

Module 2
Equations and Inequalities

Module 3
Quadratics

Module 4
Functions

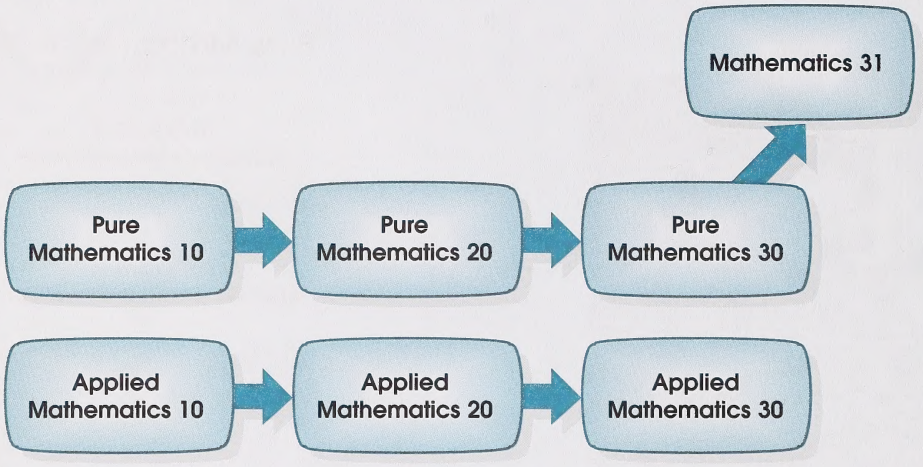
Module 5
Mathematical Reasoning and Geometry



Pure Mathematics 20 contains five modules. Work through the modules in the order given, since several concepts build on each other as you progress in the course.

High School Mathematics Courses

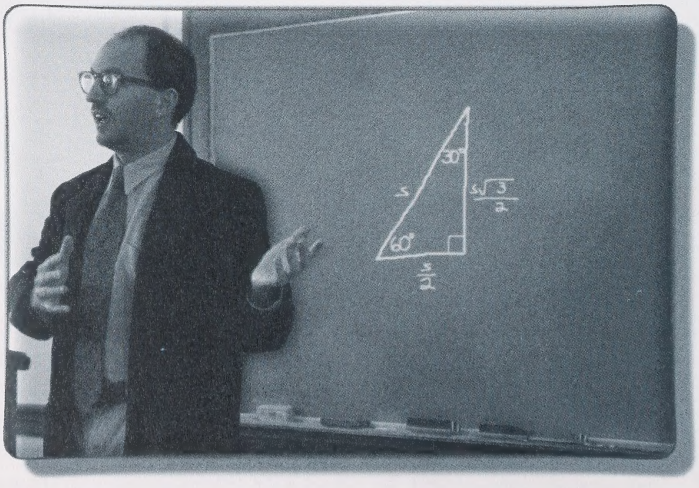
Pure Mathematics 20 is the second course in the Pure Mathematics 10–20–30 sequence of courses. Many students who take the Pure Mathematics 10–20–30 will also choose to take Mathematics 31. Another sequence of courses is Applied Mathematics 10–20–30.



How Do the Sequences Differ?

Each sequence of courses is designed for students with different mathematical strengths and interests. The Pure Mathematics 10–20–30 sequence is intended for students who are strong in mathematical theory and algebra. The Applied Mathematics 10–20–30 sequence is better suited to students who prefer to solve mathematical problems using numerical reasoning or geometry.

Each sequence of courses is designed for students with different career plans. Students who plan to take courses such as commerce, engineering, physics, and engineering technologies in post-secondary education will need to take Pure Mathematics 30 in high school. Some of these students will also need to take Mathematics 31.

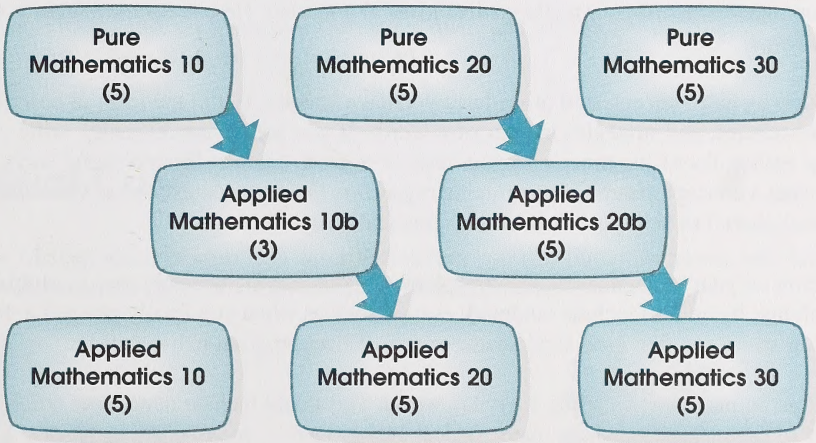


Can I Transfer to the Applied Mathematics Sequence of Courses?

You should be aware that the applied and pure mathematics courses do have some topics in common; other topics are specific to either the applied or pure mathematics courses.

Applied Topics	Common Topics	Pure Topics
• linear programming • data tables and trends • design and layout • metric and imperial measure • data presentation • vectors and matrices • periodic, fractal, and recursive patterns • financial decision making • costing and design problems	• spreadsheets • line segments and linear graphs • scaling • triangles • surveys • financial mathematics • quadratic functions • circle geometry • the bell curve	• irrational numbers • exponents • polynomial and rational expressions • mathematical expectation • growth patterns • linear and non-linear systems • operations on functions • mathematical reasoning • exponential and logarithmic functions • conics • combinations • trigonometric functions

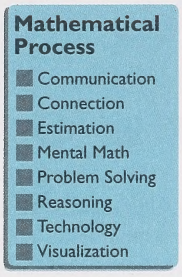
If you want to transfer from the Pure Mathematics 10–20–30 sequence to the Applied Mathematics 10–20–30 sequence at a future time, you won't have to repeat the topics that are common to pure mathematics and applied mathematics. If you decide to transfer to the Applied Mathematics sequence of courses after successfully completing Pure Mathematics 10, you may take the 3-credit course called Applied Mathematics 10b or you may take the 5-credit course called Applied Mathematics 10. If you decide to transfer after successfully completing Pure Mathematics 20, you may take the 5-credit course called Applied Mathematics 20b. The two bridging courses are shown in the following diagram:


Pure Mathematics 10 (5)
Pure Mathematics 20 (5)
Pure Mathematics 30 (5)
Applied Mathematics 10b (3)
Applied Mathematics 20b (5)
Applied Mathematics 10 (5)
Applied Mathematics 20 (5)
Applied Mathematics 30 (5)

Mathematical Processes

Throughout the course you will be expected to do the following:

- communicate mathematically
- connect mathematical ideas to everyday experiences and to concepts in other disciplines
- develop and use estimation strategies
- develop and use mental math strategies
- develop and use problem-solving strategies
- reason and justify your thinking
- select and use appropriate technologies as tools to solve problems
- use visualization to assist in processing information, making connections, and solving problems



This icon shows which mathematical process you are expected to perform.

In order to develop these mathematical processes more fully, you are encouraged to ask someone who is also taking Pure Mathematics 20 to be your study partner. You will find that having a friend to discuss mathematics with will make your studying more enjoyable.

You are also encouraged to record your mathematical ideas in a journal. Here are some suggestions for organizing your journal:

- Keep a section on new concepts and procedures you have learned. Get in the habit of describing new concepts, procedures, and strategies in your own words. If you are having difficulty, write your questions in your journal before discussing them with your teacher or study partner. Record useful ways to help you remember what a concept means. Make graphic organizers (such as context webs, Venn diagrams, or organizational charts) to help you connect mathematical ideas.

- Keep a section on your mathematical accomplishments. This can include solutions to problems that you are proud of solving. It can also include landmark events, such as when you finally grasped a difficult concept (an "aha!") or when you first used a calculator or spreadsheet program in a new way.

- Keep a section on mathematics in the everyday world. This could include newspaper articles, cartoons, and descriptions of careers that require a mathematical background.

Resources You Will Need

In order to complete Pure Mathematics 20 successfully, you will need to have access to the following resources, which can be purchased through the Learning Resources Distributing Centre:

- the *MATHPOWER™ 11* textbook, Western Edition, published by McGraw-Hill Ryerson (1999)
- a graphing calculator

  Any of the following graphing calculators may be used: *Texas Instruments* TI-83, TI-83 Plus, TI-82, TI-86, TI-89, TI-92, or TI-92 Plus; *Casio* CFX-9850Ga-Plus, CFX-9850G, CFX-9800G, or FX-9700 series; or *Sharp* EL-9600c, EL-9600, EL-9300, or EL-9200. **Note:** Some of these calculators are no longer commercially manufactured.
- a computer and a spreadsheet program

  Any of the following spreadsheet programs may be used: *ClarisWorks™ 4.0* (or higher) for Macintosh, *ClarisWorks™ 4.0* (or higher) for Windows, *Microsoft® Works 3.0* (or higher) for Windows, or *Microsoft® Access 97* and *Microsoft® Excel 97* for Windows (a part of *Microsoft® Office Professional 95* or *Microsoft® Office Professional 97*).

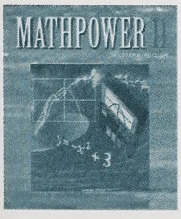
Note: Examples in this course show the TI-83 graphing calculator and the *ClarisWorks™ 4.0* spreadsheet program where applicable. If you have access to a videocassette recorder, you may wish to view the video *The TI-83 Graphing Calculator Video Tutor* to review some of the calculator's features. Check with your distance learning provider concerning the availability of this video.

Finally, you will need the following resources, which can be purchased locally:

- a mathematical instrument set (compass, protractor, ruler, and triangles)
- a binder in which to respond to the questions asked
- a binder for journal writing

Other Visual Cues

In addition to the Mathematical Process icon described earlier, you will find other visual cues throughout this course. Read the explanations given to discover what each icon prompts you to do.



Refer to the textbook.

Use the Internet to explore a topic.

Answer the questions in the Assignment Booklet.

Remember: *Any Internet website address given in this module is subject to change.*

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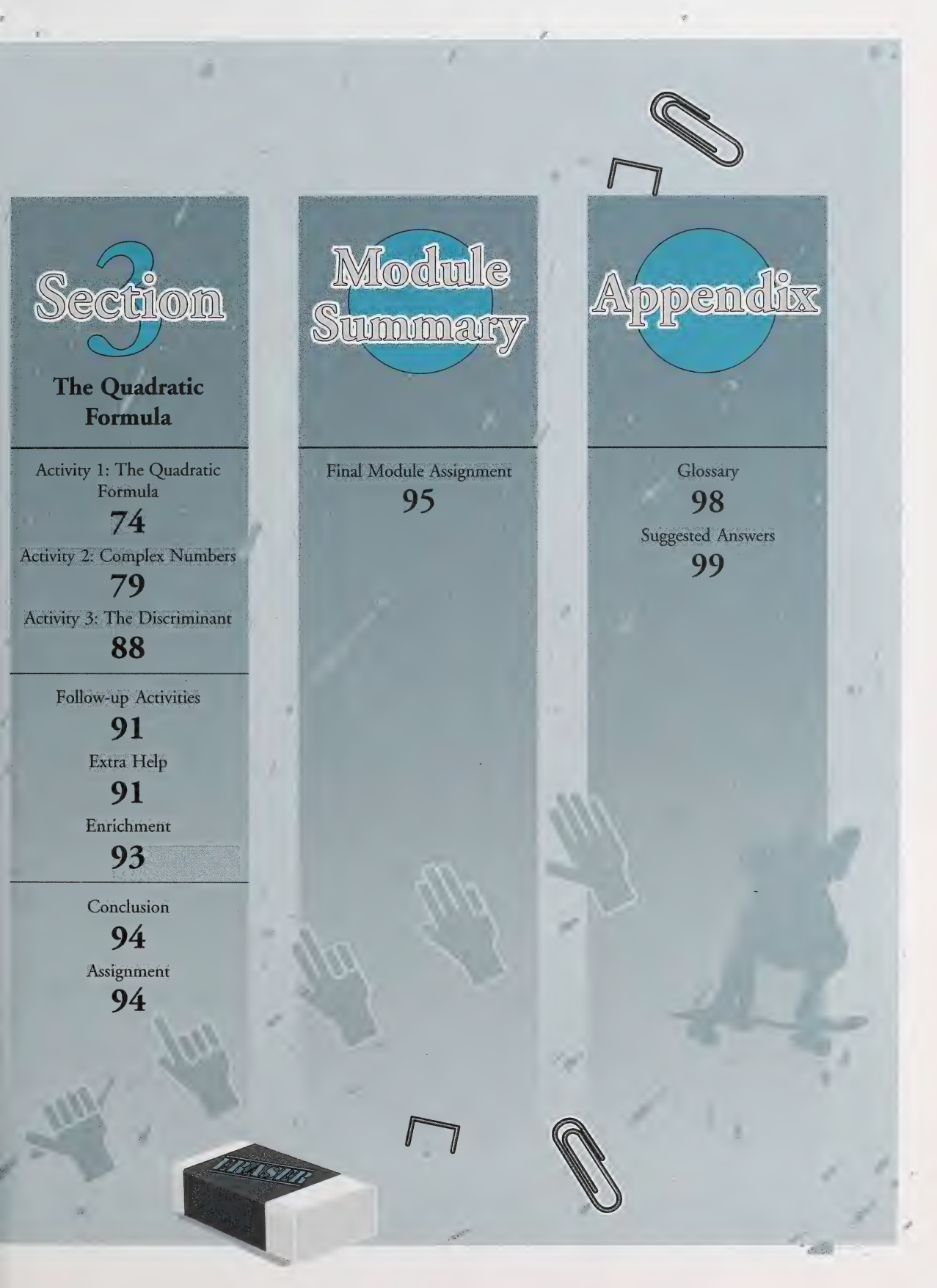

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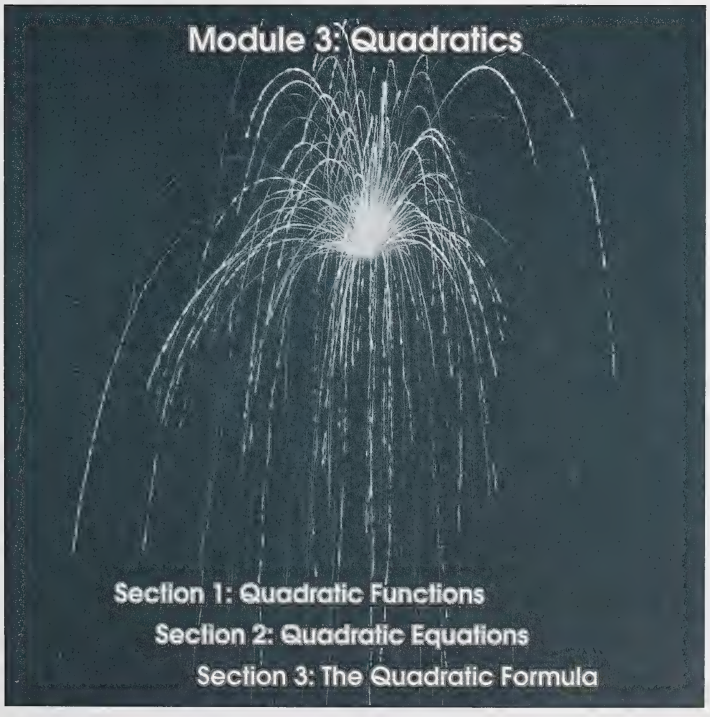
Module Overview

What do a dazzling fireworks display, the trajectory of a cannonball, the orbits of some comets, a lazy dive from a wharf on a hot summer afternoon, a shower of sparks from a welder's torch, the headlamp of a car, and a satellite dish have in common? Each event or object can be modelled by a special class of functions, called quadratics.

In this module, you will explore both quadratic functions and quadratic equations. In Section 1, you will work with quadratic functions, their graphs, and how they can be applied to everyday situations. In Sections 2 and 3, you will investigate quadratic equations—how they are related to quadratic functions and the methods you can use to solve these equations. Not only will you discover numerous applications of quadratic functions and equations in mathematics and the physical sciences, you will discover numerous applications of quadratic functions and equations in art and design.



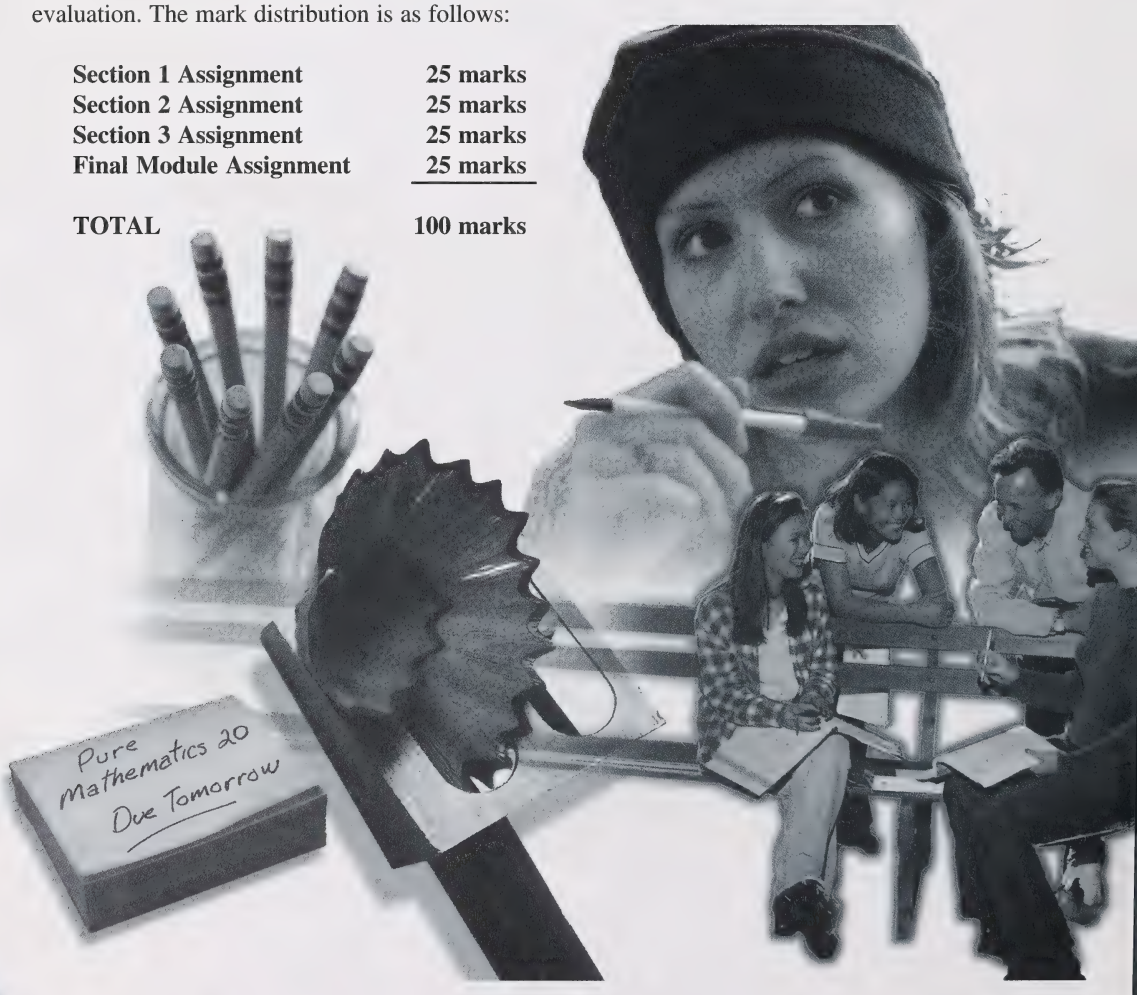
Evaluation

The document you are presently reading is called a Student Module Booklet. It will show you, step by step, what to do and how to do it.

This module, Quadratics, has three sections. Within each section, your work is grouped into activities. Within the activities, there are readings and questions for you to answer. By completing these questions you will construct your own learning, discover mathematical connections, and practise or apply what you have learned. The suggested answers in the Appendix of this Student Module Booklet will provide you with immediate feedback on your progress.

At several points in this module you will be directed to the accompanying Assignment Booklet. In this module you are expected to complete three section assignments and one final module assignment. Your grading in this module is based upon the assignments that you submit for evaluation. The mark distribution is as follows:

Section 1 Assignment	**25 marks**
Section 2 Assignment	**25 marks**
Section 3 Assignment	**25 marks**
Final Module Assignment	**25 marks**
TOTAL	**100 marks**



Strategies for Completing a Module

Before you begin, organize your materials: Student Module Booklet, textbook, Assignment Booklet, binder, lined paper, graph paper, graphing calculator, pens, pencils, and so on. Make sure you have a quiet area in which to work, away from distractions. Set up your time schedule.

To achieve success in this module, be sure to read all your instructions carefully and work slowly and systematically through the material. Remember, it's the work you do in this Student Module Booklet that will prepare you for your assignments and final test. Try to set realistic goals for yourself each day; and once you've set them, stick to them. Submit your assignments regularly, and don't forget to review your work before handing it in. Careful work habits will greatly increase your chances for success in Pure Mathematics 20.

Good luck!

Section 1

NOAO

Quadratic Functions

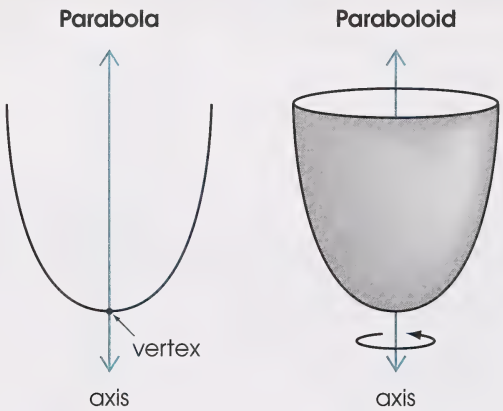
Comets in the night sky can be spectacular. The photograph on the left is of Comet Halley as it appeared in 1910. Halley is a periodic comet, returning every 76 years. Its most recent approach in 1986 was not as dramatic. Have you ever seen a comet? Do you remember seeing Comet Hale-Bopp or Comet Hyakutake in 1996?

When comets come from deep space and swing around the Sun, the orbits they trace out are parabolic (or very close to parabolic in shape). A number of unmanned space missions have been conducted or are being planned to investigate comets as they orbit the Sun. In 1985, the European Space Agency launched the Giotto Mission using an Ariane rocket. This mission was to fly by and photograph Comet Halley. On March 13, 1986, the probe came within 596 km of the comet's core. In 2003, NASA will be launching the Chamollian/ Deep Space 4 Mission. This mission will send a probe to Comet Temple 1. The probe will land on the comet's core, in late 2005 or early 2006, and then return to Earth with samples.

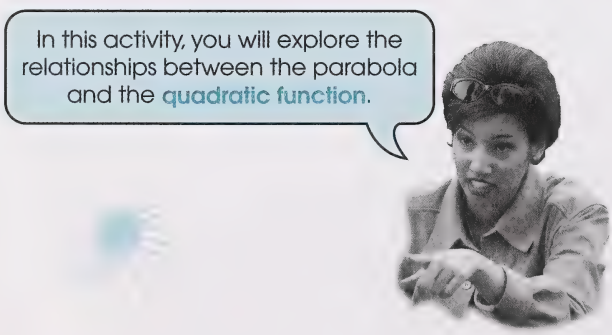
In this section, you will be studying the quadratic function. The quadratic function can be used to model the parabolic orbits of comets and a host of more down-to-earth situations, ranging from trajectories of projectiles to the optimum shape of a fenced enclosure. Specifically, you will determine the characteristics of the graphs of quadratic functions, investigate transformations of these graphs, and model real-world situations using quadratic functions.

Activity 1: Graphing $y = x^2 + q$, $y = ax^2$, and $y = ax^2 + q$

Have you looked closely at satellite dishes like the one in the photograph? The one shown is much larger than the type many homeowners have for receiving television signals. However, in spite of differences in size, the shapes of these dishes are similar. These shapes are often referred to as being parabolic. Strictly speaking, they are **paraboloids of revolution**. A paraboloid of revolution is formed by spinning a **parabola** about its **axis of symmetry**, the line that reflects the left half of the graph into the right half and vice versa.



The point of the parabola through which the axis of symmetry passes is called the **vertex**.



Mathematical Process

- [ ] Communication
- [x] Connection
- [ ] Estimation
- [ ] Mental Math
- [ ] Problem Solving
- [ ] Reasoning
- [ ] Technology
- [x] Visualization

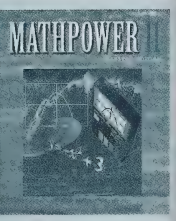
paraboloid of revolution: a three-dimensional figure formed by rotating a parabola about its axis of symmetry

parabola: a U-shaped curve obtained by graphing a quadratic function

axis of symmetry: a line that divides a curve into halves, reflecting each half into the other

vertex: the point of intersection of a parabola and its axis of symmetry

quadratic function: a function that may be written in the form $y = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$



Turn to page 104 of *MATHPOWER™ 11* and read "Graphing $y = x^2 + q$, $y = ax^2$, and $y = ax^2 + q$."

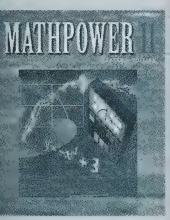
1. Answer the following questions on page 104 of the textbook:

a. "Explore: Compare the Graphs"
b. questions 1 to 5 of "Inquire"

Compare your responses with those in the Appendix, Section 1: Activity 1.

The quadratic function is a function of the form $f(x) = ax^2 + bx + c$, where the coefficients a, b, and c are real numbers and $a \neq 0$. If $a = 0$, the function would reduce to $f(x) = bx + c$, which is a linear function. In a quadratic function, there must be a squared term; the word *quadratic* means "of degree two."

The graph of a quadratic function is a parabola. You will now begin your investigation to see how changing the coefficients of the quadratic function affects its graph.



Turn to page 105 of *MATHPOWER™ 11* and read the entire page, working through Example 1.

2. Answer questions 1 to 4, 13, and 14 of "Practice" on page 109 of the textbook.

Compare your responses with those in the Appendix, Section 1: Activity 1.

To graph quadratic functions, you have several choices. You can

- set up tables of values and plot points on a grid
- use a graphing calculator
- use the graphing feature of a computer spreadsheet
- use graphing software, such as *Zap-a-Graph™*

In the following example, you will graph a quadratic function using a graphing calculator and a spreadsheet program. **Remember:** The procedures described are for the *TI-83 (Texas Instruments)* graphing calculator and *ClarisWorks™ 4.0* for Macintosh. If you are using a different graphing calculator or different spreadsheet software, you may have to modify these procedures. Please consult your manual and user's guide.

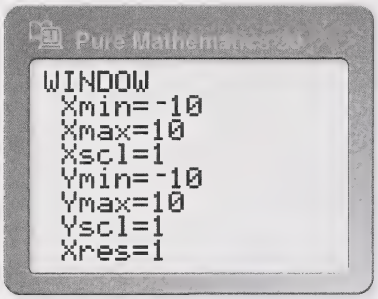
Example

Graph $y = x^2 - 4$ and give a table of values using the following:

a. a graphing calculator
b. spreadsheet software

Solution

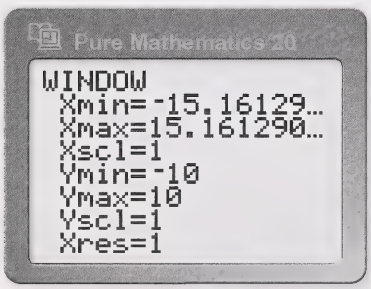
a. Before graphing the quadratic function, check your window settings. The following display shows the factory default settings.



You can obtain the factory default values by pressing ZOOM 6.

Be aware that if you use these settings, your graph will not appear exactly as it would if you used graph paper. Because the calculator screen is rectangular and not square, the graph is stretched in a horizontal direction. To compensate for this effect, press ZOOM 5. This will square up the display so the graph appears as it would on graph paper.

The new window settings are as follows:

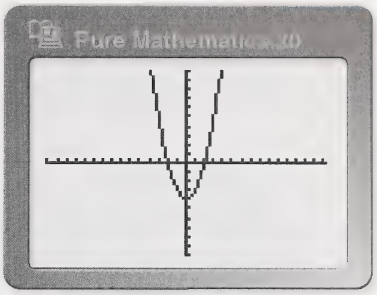


Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- ✗ Technology
- ✗ Visualization

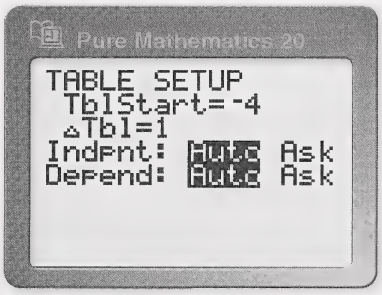
Now, enter the quadratic function, and display the graph.

Y= X,T,Θ,*n* x^2

− 4 GRAPH

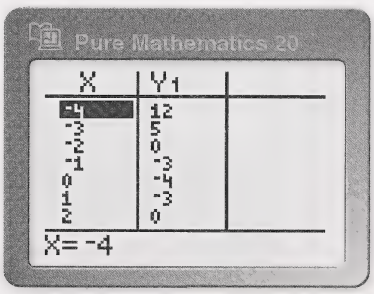


To view the table of values, first press 2nd [TBLSET].



To have your table begin with, for example, $x = -4$ and to have the values of x increase in steps of 1, enter the values as shown in the preceding display.

Now, press 2nd [TABLE] to view the table of values.



If you wish to view additional values, use the arrow keys to scroll up and down.

b. To graph the quadratic equation and display a table of values using a spreadsheet program, follow these steps:

Step 1: Open a new spreadsheet.

Step 2: Create a table of values.

- Type the headings in cell A1 and cell B1. Enter "x" in cell A1 and "y" in cell B1 (as shown). You may wish to bold the headings in order to set them apart visually from the data.

	A	B
1	x	y

- Enter the first line of the table of values.

  – In cell A2, enter "-4". This will be the first value of x for the quadratic function.

  – In cell B2, you want to find the value of y when $x = -4$. To do this, enter the formula "=A2^2-4" and press "enter" or click on the accept button (☑). ("12" will appear.)

B2	x	✓	=A2^2-4

	A	B
1	x	y
2	-4	12

- Enter the second line of the table of values.

  – In cell A3, enter "=A2+1".

  – In cell B3, enter the formula "=A3^2-4" to find the value of y and press "enter" or click on the accept button (☑). ("5" will appear.)

B3	x	✓	=A3^2-4

	A	B
1	x	y
2	-4	12
3	-3	5

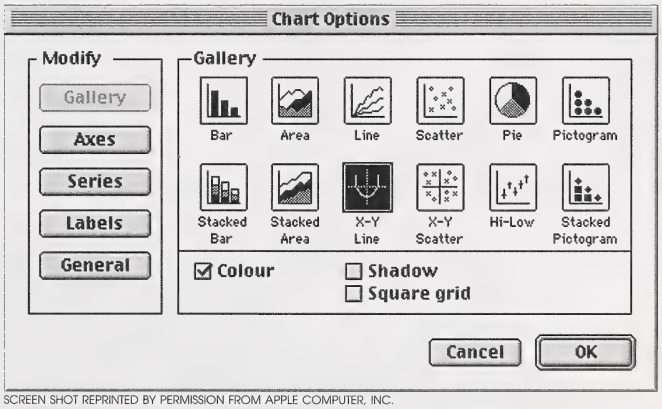
- Select cells A3 to B10. Fill in the rest of the table by clicking on the Calculate menu and selecting "Fill Down."

A3	x	√	=A2+1

	A	B
1	x	y
2	−4	12
3	−3	5
4	−2	0
5	−1	−3
6	0	−4
7	1	−3
8	2	0
9	3	5
10	4	12

Step 3: Graph the function using the table of values.

- Select cells A1 to B10.
- Click on the Options menu and select "Make Chart." The following window will appear.

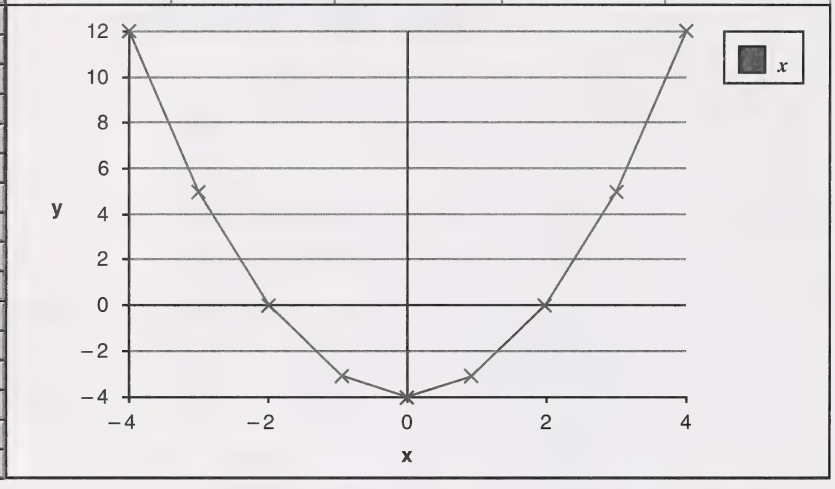


SCREEN SHOT REPRINTED BY PERMISSION FROM APPLE COMPUTER, INC.

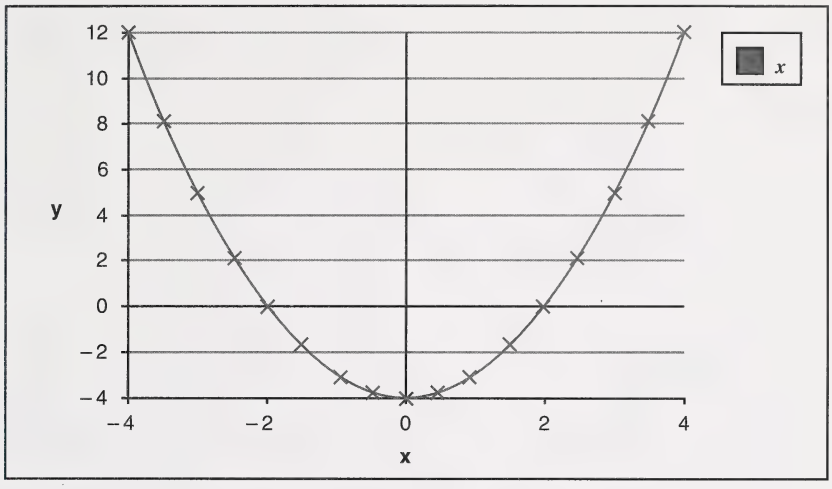
- Choose "X–Y Line"; then click "OK."

The graph of $y = x^2 - 4$ and its table of values is as follows:

	A	B	C	D	E
1	x	y			
2	−4	12			
3	−3	5			
4	−2	0			
5	−1	−3			
6	0	−4			
7	1	−3			
8	2	0			
9	3	5			
10	4	12			
11					



The graph is a series of segments joining adjacent pairs of points. To obtain a smoother curve, use a smaller interval between the x-values. For example, the following is a graph of the preceding function with an interval of 0.5 between the x-values.



Use a graphing calculator or a spreadsheet program to work through the next exercise. You will investigate how changing the value of the coefficient a in $y = ax^2$ changes the graph.

Turn to page 106 of *MATHPOWER™ 11* and work through Example 2; then read the summary that follows Example 2.

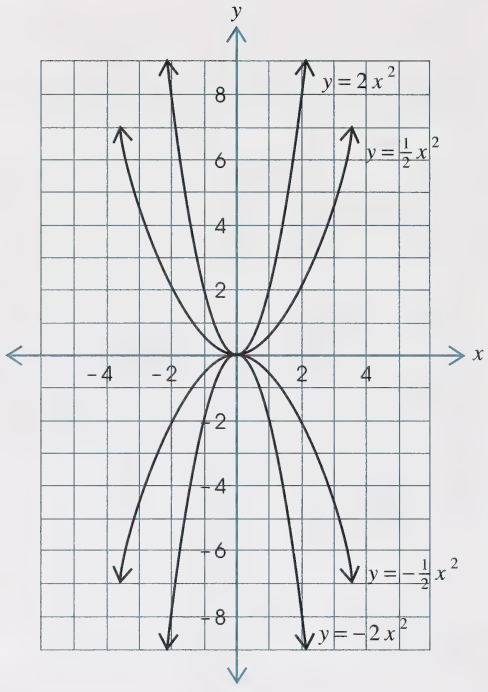
3. Answer questions 5, 6, 8, 15, 16, and 18 of "Practice" on page 109 of the textbook.

Compare your responses with those in the Appendix, Section 1: Activity 1.

As you have seen, the value of coefficient a in $y = ax^2$ determines the direction in which the parabola opens as well as the width of the curve.

Example

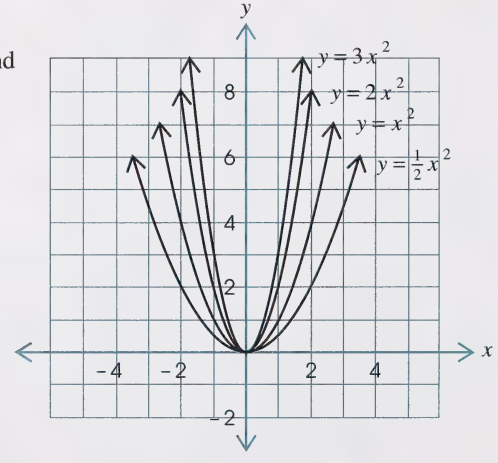
Arrange the graphs of $y = 3x^2$, $y = x^2$, $y = -2x^2$, and $y = -\frac{1}{2}x^2$ from widest to narrowest.



Solution

Recall that $y = -2x^2$ and $y = -\frac{1}{2}x^2$ are reflections of $y = 2x^2$ and $y = \frac{1}{2}x^2$, respectively, in the x-axis.

Now, compare the relative widths of the graphs of $y = 3x^2$, $y = x^2$, $y = 2x^2$, and $y = \frac{1}{2}x^2$.



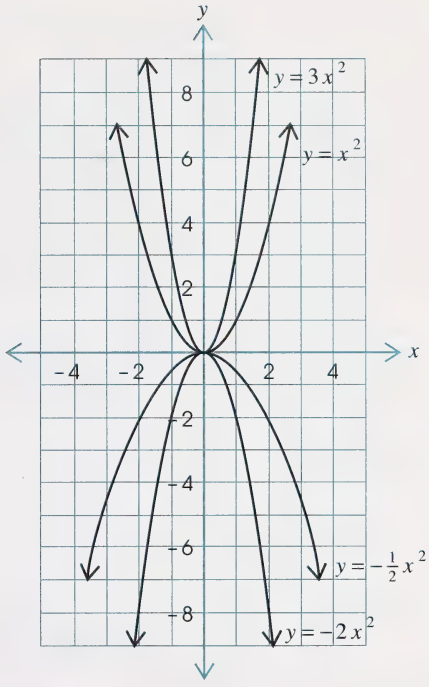
Earlier, you investigated the graphs of functions of the form $y = ax^2$. You discovered that for positive a values, the larger a is, the narrower the parabola is. Therefore, when you arrange the graphs from widest to narrowest, the functions appear in the following order: $y = \frac{1}{2}x^2$, $y = x^2$, $y = 2x^2$ and $y = 3x^2$.

Because the graphs of $y = -\frac{1}{2}x^2$ and $y = \frac{1}{2}x^2$ are the same width and the graphs of $y = -2x^2$ and $y = 2x^2$ are the same width, the graphs, arranged from the widest to the narrowest, are $y = -\frac{1}{2}x^2$, $y = x^2$, $y = -2x^2$, and $y = 3x^2$.

The coefficients $-\frac{1}{2}$, 1, -2, and 3 appear in ascending order when their absolute values are compared.

$$\left|-\tfrac{1}{2}\right| < |1| < |-2| < |3|$$

Remember: The larger $|a|$ is in $y = ax^2$, the narrower the graph is.



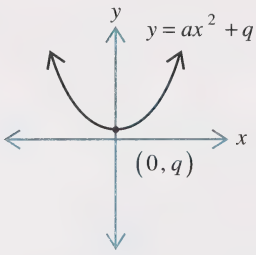
You have just explored graphs of the form $y = ax^2$ and $y = x^2 + q$. Now, you will study the combined effects of coefficients a and q in a graph of the form $y = ax^2 + q$.

Turn to page 107 of *MATHPOWER™ 11* and work through Examples 3 and 4.

4. Answer questions 9, 10, 17, 19, 21, 25, 48, 49, 54, 57, and 61 of "Practice" on pages 109 and 110 of the textbook.

Compare your responses with those in the Appendix, Section 1: Activity 1.

The y-intercept of the graph of $y = ax^2 + q$ is $(0, q)$.



Here, $a > 0$ and $q > 0$.

To determine the x-intercepts, if any, you can use algebraic methods or your graphing calculator.

Mathematical Process
Communication
Connection
Estimation
Mental Math
Problem Solving
Reasoning
Technology
Visualization

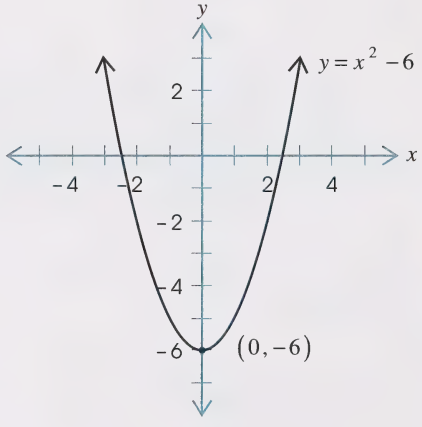
Example

Sketch the graph of $y = x^2 - 6$. State the x-intercepts if any.

Solution

Method 1: Using Algebra

The vertex of the graph of $y = x^2 - 6$ is $(0, -6)$. Because the graph opens upward, the graph crosses the x-axis at two points.



To find the x-intercepts, let $y = 0$.

$\therefore\ x^2 - 6 = 0$

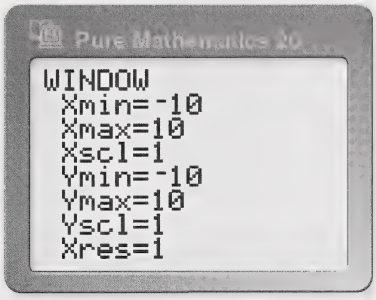
$x^2 = 6$

$x = \pm\sqrt{6}$ (or approximately ± 2.45)

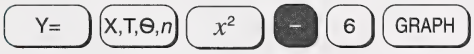
The graph crosses the x-axis at $\left(-\sqrt{6}, 0\right)$ and $\left(\sqrt{6}, 0\right)$.

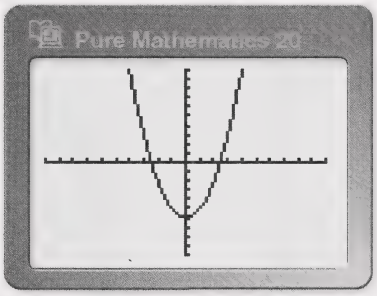
Method 2: Using a Graphing Calculator and the Trace Feature

Use the default window settings.



To graph the function $y = x^2 - 6$, press the following:

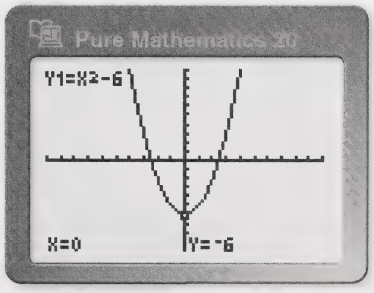




The graph appears to cross the x-axis between 2 and 3 and between -2 and -3.

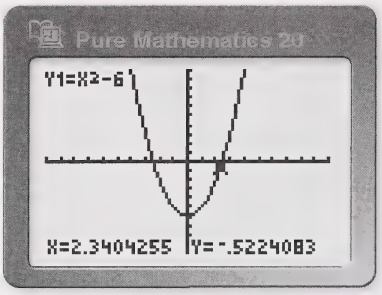
The Trace feature can be used to obtain initial approximations of the x-intercepts. Press

. The following display should appear.



Note: The coordinates at the bottom of the screen give the location of the cursor.

Position the cursor as close as possible to one of the x-intercepts. For example, move the cursor to the x-intercept between 2 and 3 using the arrow keys.

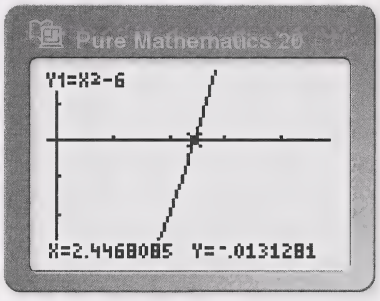


Note: When using the Trace feature, only the right- and left-arrow keys will move the cursor.

Notice that the preceding display shows the cursor is located at $x = 2.340\,425\,5$ on the curve. This is not the x-intercept because $y = -0.522\,408\,3$, not 0. If you press the right arrow key once more, the cursor will move above the x-axis to $x = 2.553\,191\,5$ and $y = 0.518\,786\,78$.

Now, you know that an x-intercept occurs between 2.34 and 2.55, a closer approximation.

To obtain an even closer approximation, you can zoom in on that part of the graph. Press ZOOM 2 ENTER TRACE. Again, you can obtain an approximation by moving the cursor as close to the x-intercept as possible.



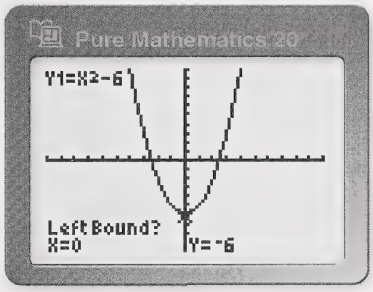
An approximation for this x-intercept is 2.45. Because the axis of symmetry is $x = 0$, the other x-intercept is approximately -2.45.

Method 3: Using a Graphing Calculator and the Zero Feature

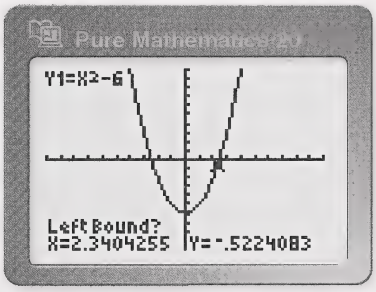
Because the y-value at an x-intercept of a graph of a function is zero, the value of the x-intercept is called a **zero** of the function.

Note: If a is a zero of function f, then $f(a) = 0$.

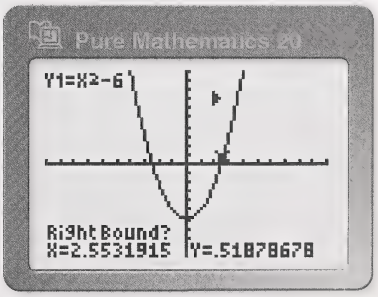
Graph the function as in Method 2; then press 2nd [CALC] 2. The following display should appear.



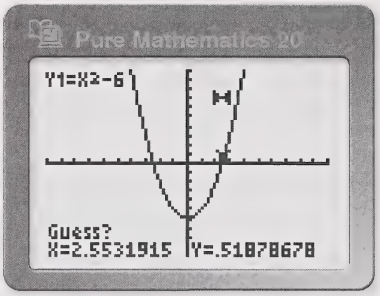
Using the arrow keys, move the cursor along the curve to a point just left of the x-intercept and press ENTER. This gives you a left bound for the zero. In the display shown, the left bound selected is $x = 2.340\,425\,5$.



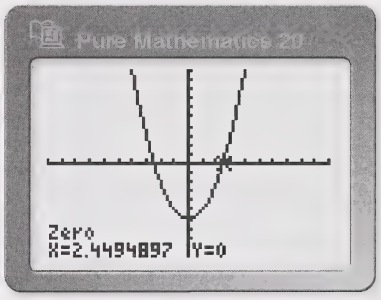
The calculator then prompts you for a right bound. Move the cursor just right of the x-intercept and press ENTER. In the display shown, the right bound selected is $x = 2.553\,191\,5$.



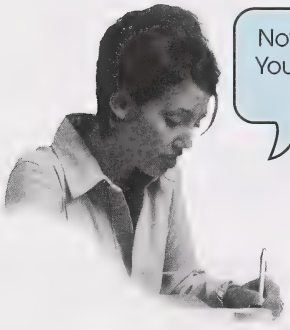
Now, the calculator prompts you for your guess. Simply press the left arrow key once and press ENTER.



The calculator now shows that $x = 2.449\,489\,7$, correct to 7 decimal places, is a zero of the function and, therefore, an x-intercept of the graph.



Therefore, because the axis of symmetry is $x = 0$, the x-intercepts are approximately ± 2.45.



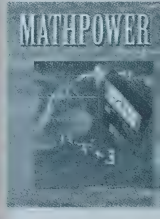
Now, put your newly acquired skills to work! You will look at a number of applications of the quadratic function.

Turn to page 108 of *MATHPOWER™ 11* and work through Example 5.

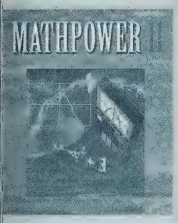
5. Answer the following questions on pages 109 to 111 of the textbook:

a. questions 28, 30, 34, 36, and 43 of "Practice"
b. questions 64, 65, 67 to 71, 72.a., and 76 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 1.



Mathematical Process
- Communication
- [X] Connection
- Estimation
- Mental Math
- Problem Solving
- [X] Reasoning
- Technology
- Visualization



Now Try This

In this activity, you investigated transformations of the basic quadratic function $y = x^2$. You will discover that the transformations of the absolute value function, $y = |x|$, are produced the same way. As a matter of fact, transformations of more general functions can be determined by thinking of $y = x^2$.

6. Turn to "INVESTIGATING MATH" on pages 102 and 103 of *MATHPOWER™ 11* and answer the following:

a. questions 1 to 6 of Investigation 1, "Exploring $y = |x|$"
b. questions 1 to 5 of Investigation 2, "Comparing $y = |x|$ and $y = |x| + q$"
c. questions 1 to 6 of Investigation 5, "Comparing $y = |x|$ and $y = a|x|$"

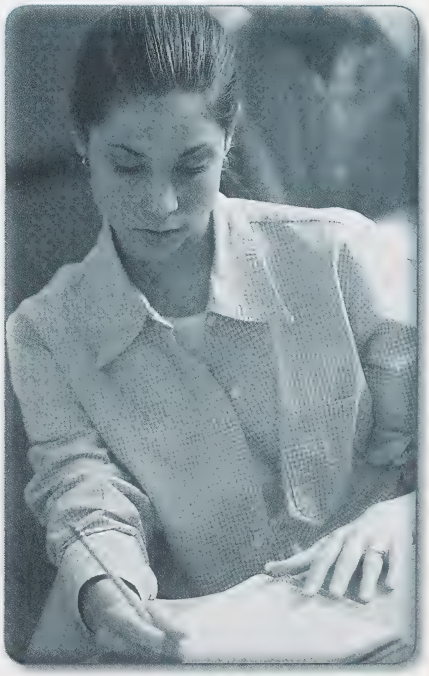
Compare your responses with those in the Appendix, Section 1: Activity 1.

Looking Back

Mathematical Process
- [X] Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

In this activity, you were introduced to the quadratic function $y = ax^2 + bx + c$ and its graph. You investigated transformations of the graph of the basic quadratic function $y = x^2$. In particular, you studied the effects of a and q in $y = ax^2 + q$. You determined the domains and ranges, axes of symmetry, vertices, and x- and y-intercepts of these functions using both algebraic techniques and technology.

Before you start working on the next activity, list in your journal all of the everyday objects you can think of that incorporate the shape of the parabola. Don't forget to include parabolic mirrors and headlamps.



Activity 2: Graphing $y = a(x - p)^2 + q$

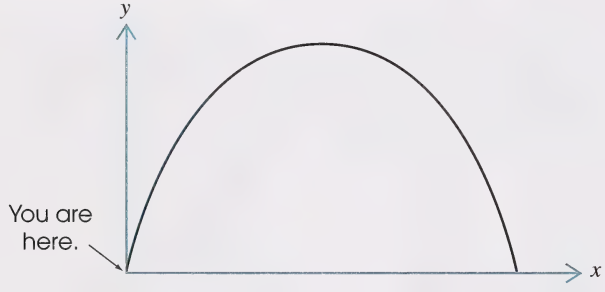
During professional baseball's 1998 regular season, Mark McGwire of the St. Louis Cardinals hit 70 home runs. Both Mark McGwire and Sammy Sosa, Chicago Cubs' outfielder who hit 66 homers, broke Roger Maris's single-season record of 61 home runs—a record that had lasted since 1961.

CORBIS/AFP

Mark McGwire's 70th home-run ball

The trajectories of baseballs, and other projectiles, are approximate parabolas. Even if a ball is hit or thrown vertically, its height above the ground can be modelled using a quadratic function.

Suppose Mark McGwire hit a baseball and watched it arc and strike the ground some distance away. The height of the ball, y, can be mapped as a function of the distance, x, it travels horizontally. If his position is fixed at the origin, then the parabolic path has a vertex and axis of symmetry that are not along the vertical axis.



In this activity, you will explore the transformations of the basic quadratic function, $y = x^2$, which will produce curves of this type.

Mathematical Process

- Communication
- [X] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

Mathematical Process

- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- [X] Visualization

Turn to page 114 of *MATHPOWER™ 11* and read "Graphing $y = a(x-p)^2 + q$."

1. Answer the following questions on pages 114 and 115 of the textbook:

a. "Explore: Compare the Graphs"
b. questions 1 to 6 of "Inquire"

Compare your responses with those in the Appendix, Section 1: Activity 2.

Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- X Technology
- Visualization

In the preceding questions, you located vertices and axes of symmetry by analysing transformations of the basic function $y = x^2$. You can also determine vertices and axes using a graphing calculator.

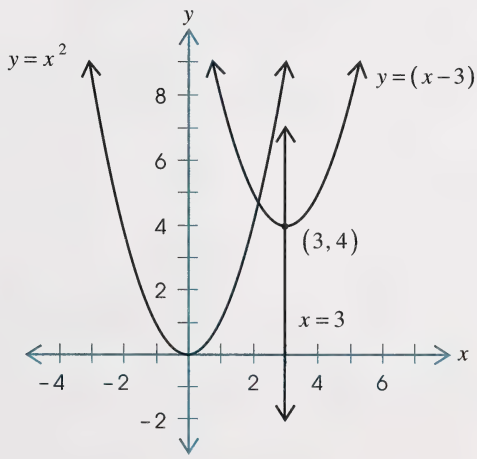
Example

Determine the vertex and axis of symmetry of $y = (x-3)^2 + 4$ by

a. analysing transformations of the basic function $y = x^2$
b. using a graphing calculator and its Maximum feature or Minimum feature

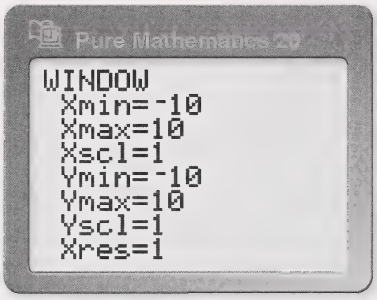
Solution

a. The graph of $y = (x-3)^2 + 4$ is the graph of $y = x^2$ translated 3 units to the right and 4 units upward.

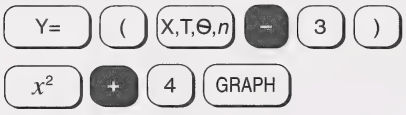


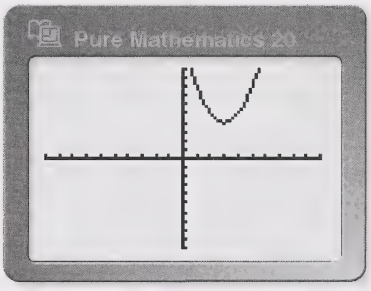
The vertex is $(3, 4)$, and the axis of symmetry is $x = 3$.

b. Set the display so it shows the standard window settings.



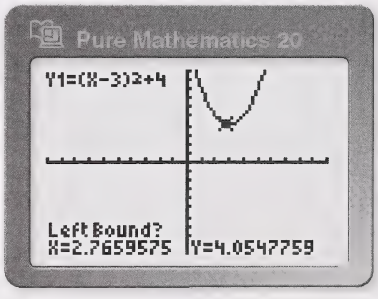
Graph the function $y=(x-3)^2+4$ by pressing the following:





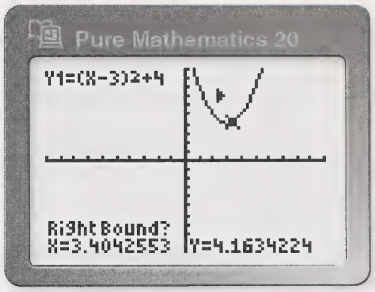
Because the graph opens upward, this function has a minimum value. Therefore, you will use the Minimum feature from the CALCULATE menu. To determine the minimum, do the following:

Press 2nd [CALC] 3. This procedure displays the graph with a movable cursor. Use the arrow keys to move the cursor along the graph until it is close to but still to the left of the vertex. (In the display, the cursor is positioned at $x=2.765\,957\,5$, $y=4.054\,775\,9$.)

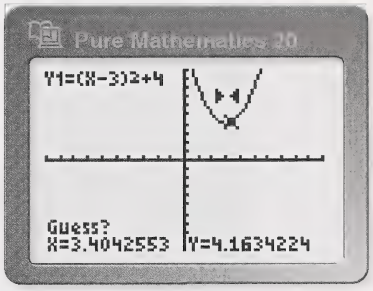


The screen prompts you to enter this point as a left bound. Press ENTER to accept.

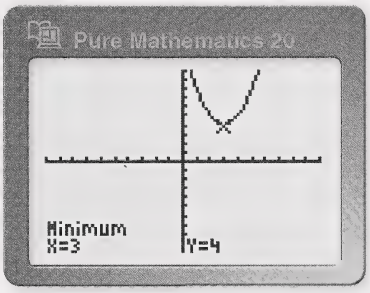
You are now prompted to enter a right bound. Use the arrow keys to position the cursor to the right of the vertex. (In the display, the cursor is positioned at $x = 3.404\,255\,3$, $y = 4.163\,422\,4$.) Press ENTER.



You are now prompted to guess. Simply use the arrow keys to position the cursor between the left bound and right bound. Simply press the left arrow key once and then ENTER.



The following display should appear.

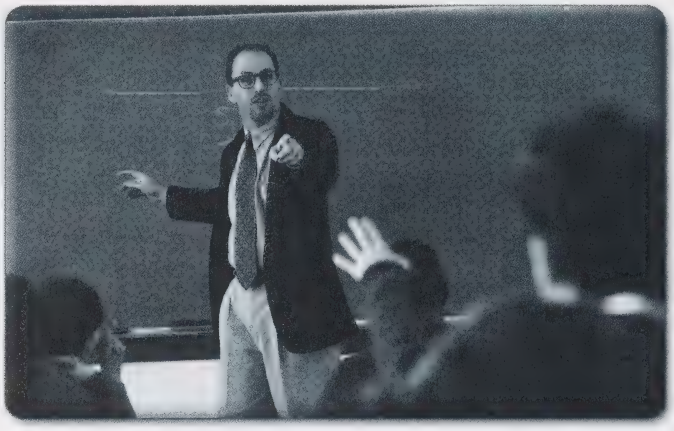


Therefore, the vertex is $(3, 4)$, and the axis of symmetry is $x = 3$.

2. Repeat the preceding example using $y = -(x-2)^2 - 1$.

Compare your responses with those in the Appendix, Section 1: Activity 2.

Now, you will further explore the **standard form of a quadratic function**—a quadratic equation written in the form of $y = a(x - p)^2 + q$, where a, p, and q are real numbers and $a \neq 0$. You will compare its graph with that of the basic function $y = x^2$ and describe the translations, in terms of a, p, and q. You will determine the vertex, the axis of symmetry, the maximum or minimum value, the domain, and the range of the graph of the quadratic function.



Turn to page 115 of *MATHPOWER™ 11* and read from the red line to the red line near the top of page 117, working through Examples 1 and 2.

3. Answer questions 1, 2, 4, 5, 13, 14, 23, and 24 of "Practice" on page 118.

Compare your responses with those in the Appendix, Section 1: Activity 2.

The *x*-value of the vertex, the horizontal translation, and the axis of symmetry can all be determined the same way from the standard form of the quadratic function $y = a(x - p)^2 + q$.

Example

Determine the vertex, the horizontal translation, and the axis of symmetry of $y = 2(x+5)^2 + 6$.

Solution

Compare the standard form of a quadratic function with the equation of the graph.

$$y = a(x-p)^2 + q \qquad y = 2(x+5)^2 + 6$$

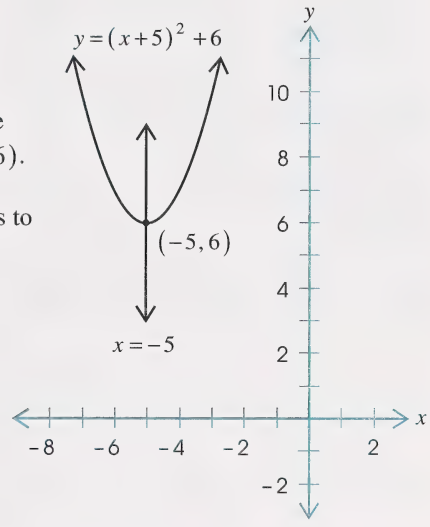
$\therefore\ -p = 5$ and $q = 6$

$= -5$

The x-value of the vertex is -5, and the y-value of the vertex is 6; therefore, the vertex is $(-5, 6)$.

The horizontal translation of the graph is 5 units to the left.

The axis of symmetry is $x = -5$.



Mathematical Process

- [ ] Communication
- [x] Connection
- [ ] Estimation
- [ ] Mental Math
- [ ] Problem Solving
- [x] Reasoning
- [ ] Technology
- [x] Visualization

You can also determine the axis of symmetry by simply setting the expression $x - p$ equal to 0 and then solving for x.

In the preceding example, $x + 5 = 0$ gives you $x = -5$, which is the axis of symmetry, the x-value of the vertex, and the horizontal translation.

Now, you will look at the intercepts of the graphs of quadratic functions, determine a quadratic function from a description of its graph, and solve a number of real-world applications.

Turn to pages 117 and 118 of *MATHPOWER™ 11* and work through Examples 3 to 5.

4. Answer the following questions on pages 119 to 121 of the textbook:

a. questions 38, 40, 48, 52, 55, 59, and 64 of "Practice"
b. questions 71 to 75, 79, 81, 86, and 88 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 1: Activity 2.

Now Try This

You will now compare the transformations of the absolute value function $y = a|x - p| + q$ with the transformations of the quadratic function $y = a(x - p)^2 + q$.

5. Turn to "INVESTIGATING MATH" on pages 102 and 103 of *MATHPOWER™ 11* and answer the following:

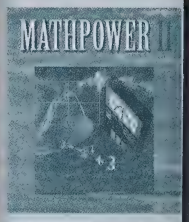
a. questions 1 to 4 of Investigation 3, "Comparing $y = |x|$ and $y = |x - p|$"
b. questions 1 to 3 of Investigation 4, "Comparing $y = |x|$ and $y = |x - p| + q$"
c. questions 4 and 8 of Investigation 6, "Combining Transformations"

Compare your responses with those in the Appendix, Section 1: Activity 2.

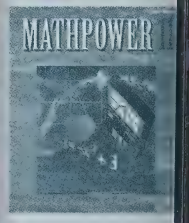
Looking Back

In this activity, you explored the standard form of the quadratic function, $y = a(x - p)^2 + q$. You compared its graph with that of the basic function, $y = x^2$, and described the translations in terms of a, p, and q. You then determined the vertex, the axis of symmetry, the maximum or minimum value, the domain and range, and the x- and y-intercepts of a graph of a quadratic function. Finally, you determined the equations of quadratic functions from their graphs or from descriptions of their graphs and applied your knowledge of the standard form of the quadratic function to problems and real-world situations.

A friend has telephoned you about a quadratics assignment. You are trying to describe a particular graph. Without giving the equation of the function, what is the least information you could give your friend so he or she can determine the exact equation? How many different ways can you do this? Record your ideas in your journal.



Mathematical Process
- Communication
- [X] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization



Mathematical Process
- [X] Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

Activity 3: Graphing $y = ax^2 + bx + c$ by Completing the Square

Mathematical Process
- Communication
- [X] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

One of the challenges of spaceflight is coping with the effects of zero gravity. The NASA astronaut in the photo has "stepped off" the shuttle and is flying free of the spacecraft. Have you ever wondered what it would be like to experience weightlessness?

NASA

The effects of weightlessness can be duplicated for short periods inside an aircraft flying along a parabolic arc. The passengers inside the aircraft travel the same trajectory and are "weightless" relative to the cabin of the aircraft.

In 1994 and 1995, groups of students, through the sponsorship of the European Space Agency, were given the opportunity to fly in NASA's KC-135 aircraft and experience weightlessness first-hand. If you have access to the Internet, you can find out more about these students' once-in-a-lifetime experience.

http://esapub.esrin.esa.it/bulletin/bullet85/ocke85.htm

The parabolic path of the KC-135 can be described by a quadratic function in **general form**, $f(x) = ax^2 + bx + c$. In the previous activities, you investigated quadratic functions in standard form, $f(x) = a(x - p)^2 + q$. In this activity, you will transform quadratic equations from general form to standard form.

Turn to page 126 of *MATHPOWER™ 11* and read "Graphing $y = ax^2 + bx + c$ by Completing the Square."

1. Answer the following questions on pages 126 and 127 of the textbook:

a. "Explore: Look for a Pattern"
b. questions 1 and 2 of "Inquire"

Compare your responses with those in the Appendix, Section 1: Activity 3.

As you have just seen, to determine the constant that forms a trinomial square when added to $x^2 + bx$, divide the **coefficient** of x by 2 and square the result.

$$\left(\frac{b}{2}\right)^2 = \frac{b^2}{4}$$

The resulting trinomial square may be written as the square of a binomial.

$$x^2 + bx + \frac{b^2}{4} = \left(x + \frac{b}{2}\right)^2$$

Example

Determine the constant term that completes the following square; then factor the result.

$$x^2 + 10x + \square$$

Solution

Divide the coefficient of x, 10, by 2.

$$10 \div 2 = 5$$

Square the quotient.

$$5^2 = 25$$

The constant 25 completes the square.

$$\therefore\ x^2 + 10x + 25 = (x+5)^2$$

Now, look at why **completing the square** is part of the process of converting a quadratic function from general form to standard form.

completing the square: the process of transforming a quadratic function from general form to standard form

Turn to page 127 of *MATHPOWER™ 11* and read from the red line to the bottom of the page, working through Example 1.

2. Answer questions 1, 5, 7, 9, 15, 16, 18, 19, 24, and 26 of "Practice" on page 131.

Compare your responses with those in the Appendix, Section 1: Activity 3.

In all the exercises you have completed so far, the quadratic functions were of the form $y = ax^2 + bx + c$, where $a = 1$.

You will now explore completing the square for functions of the type $y = ax^2 + bx + c$, where $a \neq 1$ (for example, $y = 3x^2 - 12x + 11$).

Turn to pages 128 and 129 of *MATHPOWER™ 11* and work through Examples 2 to 4.

3. Answer questions 28, 33, 36, 38, 47, 49, 52, 54, 58, and 66 of "Practice" on page 131.

Compare your responses with those in the Appendix, Section 1: Activity 3.

In many maximum and minimum problems, you are not given the quadratic function. You must form the quadratic that models the problems. As you work through the following example, reflect on the steps of the problem-solving process.

Mathematical Process
- [x] Communication
- [x] Connection
- [ ] Estimation
- [ ] Mental Math
- [x] Problem Solving
- [x] Reasoning
- [x] Technology
- [ ] Visualization

Example

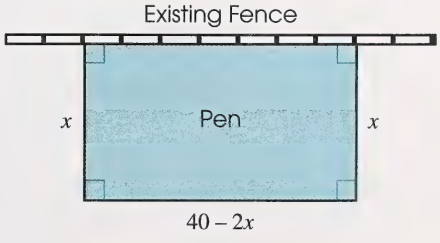
Samantha is helping her mother plan and construct a rectangular pen using 40 m of fencing. They decide to build the enclosure along an existing fence. If they want the fence to enclose a maximum area, what should be the dimensions of the pen?

Solution

Method 1: Using Algebra

Step 1: Understand the Problem

Samantha and her mother begin by sketching the pen. They observe that the 40 m of fencing will be used on three sides only, since the existing fence will be the fourth side of the rectangle.



They represent the width of the pen with a variable.

"Let x be the width."

Because there are two widths, one on the left and one on the right, they note that $2x$ will be used to fence those sides. Because they have 40 m of fencing, there will be $40-2x$ for the third side.

"Therefore, the length is $40-2x$."

Samantha writes the formula for area and points out that it is the area, A, that must be a maximum.

$$\begin{aligned} A &= \text{length} \times \text{width} \\ &= (40-2x)x \end{aligned}$$

Step 2: Think of a Plan

Samantha and her mother observe that the area formula is a quadratic function. They know that maximum values of quadratic functions, if they exist, can be obtained from the standard form.

Step 3: Carry Out the Plan

Convert the quadratic function to standard form in order to find the maximum area and the width of the pen.

$$\begin{aligned} A &= (40-2x)x \\ &= 40x-2x^2 && \text{Expand.} \\ &= -2x^2+40x && \text{Express the function in general form.} \\ &= -2\left(x^2-20x\right) && \text{Factor the coefficient of } x^2. \\ &= -2\left[\left(x^2-20x+100\right)-100\right] && \text{Complete the square inside the brackets.} \\ &= -2\left[(x-10)^2-100\right] && \text{Factor the perfect square trinomial.} \\ &= -2(x-10)^2+200 && \text{Expand.} \end{aligned}$$

Therefore, the maximum area is 200 m^2 and occurs when the width of the pen is 10 m.

If $x=10$, then

$$\begin{aligned} \text{length} &= 40-2x \\ &= 40-2(10) \\ &= 40-20 \\ &= 20 \text{ m} \end{aligned}$$

Step 4: Look Back

Samantha and her mother check the dimensions to see if they yield the same area as was obtained from the function.

$$\begin{aligned} A &= \text{length} \times \text{width} \\ &= 20(10) \\ &= 200 \text{ m}^2 \end{aligned}$$

They also check and see if other dimensions yield a greater area. They choose values of x above and below $x = 10$.

If $x = 9$, then

$$\begin{aligned} A &= 40x - 2x^2 \\ &= 40(9) - 2(9)^2 \\ &= 198 \text{ m}^2 \end{aligned}$$

If $x = 11$, then

$$\begin{aligned} A &= 40x - 2x^2 \\ &= 40(11) - 2(11)^2 \\ &= 198 \text{ m}^2 \end{aligned}$$

Since neither $x = 9$ nor $x = 11$ yield a greater area, the dimensions of the pen should be 10 m × 20 m.

Method 2: Using a Spreadsheet

You can use a spreadsheet to list and compare pen areas for different possible dimensions. From the spreadsheet, the maximum area can be selected and checked against the results of Method 1.

Step 1: Open a new spreadsheet, and enter the headings as shown.

	A	B	C
1	Width, x	Length, 40 – 2x	Area

Step 2: Complete the first row of values, starting with a width of 1 m.

- In cell A2, enter "1".
- In cell B2, enter the formula "=40-2*A2".
- In cell C2, enter the formula "=A2*B2".

The spreadsheet automatically calculates the length and the area as shown.

	A	B	C
1	Width, x	Length, 40 – 2x	Area
2	1	38	38

Step 3: Define the entries in Row 3 so that they are related to the entries in Row 2.

- In cell A3, enter "=A2+1". This will calculate the new width as 2 m.
- In cell B3, enter the formula "=40-2*A3".
- In cell C3, enter the formula "=A3*B3".

The spreadsheet automatically calculates the length and area as shown.

	A	B	C
1	Width, x	Length, 40 – 2x	Area
2	1	38	38
3	2	36	72

Notice that the area begins to increase as the width increases.

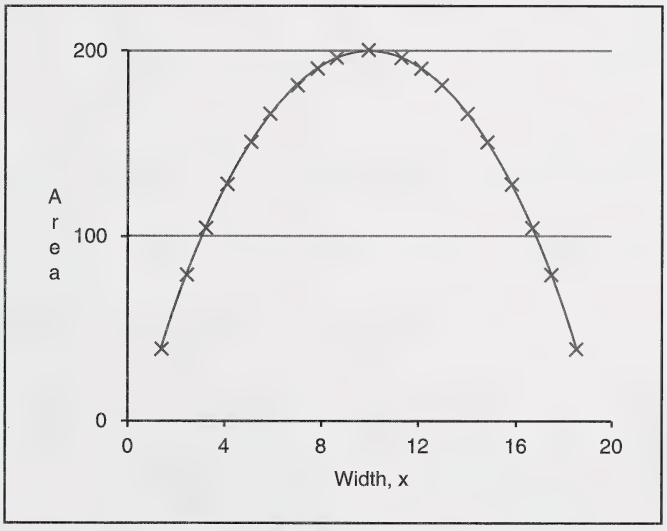
Step 4: Highlight cells A3 to C20; then click on the Calculate menu, and select "Fill Down."

	A	B	C
1	Width, x	Length, 40 – 2x	Area
2	1	38	38
3	2	36	72
4	3	34	102
5	4	32	128
6	5	30	150
7	6	28	168
8	7	26	182
9	8	24	192
10	9	22	198
11	10	20	200
12	11	18	198
13	12	16	192
14	13	14	182
15	14	12	168
16	15	10	150
17	16	8	128
18	17	6	102
19	18	4	72
20	19	2	38

According to the spreadsheet, the maximum area is $200\ \text{m}^2$ and occurs when the dimensions of the pen are $10\ \text{m} \times 20\ \text{m}$.

You can use the graph of the function to observe what happens to the area as the width increases.

To obtain this graph, delete columns B and C. Then in cell B1, enter the heading "Area." In cell B2, enter the formula "=A2*(40-2*A2)". Highlight cells B2 to B20 and fill down. Finally, highlight columns A and B; select "Make Chart" from the Options menu; choose X–Y Line; and click "OK."



From the graph, you can see that a maximum area of 200 occurs when $x = 10$.

Turn to page 130 of *MATHPOWER™ 11* and work through Examples 5 and 6.

4. Answer questions 70.a., 70.c., 71, 73, 74, 75, 80, 81, 82, 85, 87, 92, and 93 of "Applications and Problem Solving" on pages 132 to 134.

Compare your responses with those in the Appendix, Section 1: Activity 3.

Now Try This

As you have seen, you can use quadratic functions to model maximum and minimum problems in geometry. Often, you can use carefully drawn diagrams to check if your solutions are reasonable. You will be using a diagram to help you answer the following question.

5. Turn to page 134 of *MATHPOWER™ 11* and answer question 88 of "Applications and Problem Solving."

Compare your response with the one in the Appendix, Section 1: Activity 3.

Looking Back

In this activity, you transformed quadratic functions from general form to standard form by completing the square. Once the functions were in standard form, you sketched and described their graphs. In particular, you determined the minimum or maximum values of these functions. In a number of applications, you determined the equations of the functions that modelled the problems first, then completed the square to find extreme values.

Select one of the maximum or minimum word problems that you solved by completing the square. Solve that problem using a spreadsheet, and write up a detailed solution in your journal. Give the question to a friend in Pure Mathematics 20 to try.

Follow-up Activities

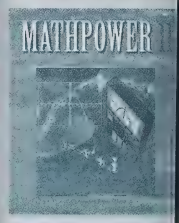
If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

In this section, you sketched and described quadratic functions by analysing them in standard form, $y = a(x - p)^2 + q$. If the functions were not in standard form, you converted them to standard form by using a procedure called **completing the square**. This procedure is called completing the square because you form trinomial squares as part of the process.

Mathematical Process

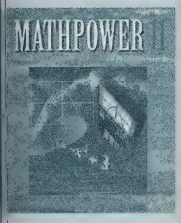
- Communication
- [X] Connection
- Estimation
- Mental Math
- [X] Problem Solving
- Reasoning
- Technology
- [X] Visualization



Mathematical Process

- [X] Communication
- [X] Connection
- Estimation
- Mental Math
- [X] Problem Solving
- Reasoning
- [X] Technology
- Visualization

Mathematical Process
- Communication
- [x] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- [x] Visualization



Completing the square can be modelled using algebra tiles. You can use the algebra tiles to represent the terms in the trinomial and arrange them to form a geometric square. If you can visualize completing the square using algebra tiles, you will become more skilled in a procedure that is fundamental to all three sections of this module.

Turn to "INVESTIGATING MATH" on page 125 of *MATHPOWER™ 11* and answer the following:

1. questions 1 and 2 of Investigation 1, "Making Squares"
2. questions 1 to 6 of Investigation 2, "Completing the Square"

Compare your responses with those in the Appendix, Section 1: Follow-up Activities, Extra Help.

Enrichment

Example

Determine the linear function from its partial table of values.

x	0	1	2	3
y	3	5	7	9

Solution

x	0	1	2	3
y	3	5	7	9

Δy → 2 2 2

In this case, the change in y (written Δy) is 2 for a change in x (written Δx) of 1.

$$\therefore \text{ slope, } m = \frac{\Delta y}{\Delta x}$$
$$= \frac{2}{1}$$
$$= 2$$

or $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

The slope is the difference between consecutive y-values in the table.

From the point $(0, 3)$, the value of $b = 3$.

Substitute $m = 2$ and $b = 3$ into $y = mx + b$ to obtain the linear function $y = 2x + 3$.

Is there a similar relationship between the coefficients of $y = ax^2 + bx + c$ and its partial table of values?

Example

Explore the partial table of values of $y = 3x^2 - 5x + 7$.

Solution

x	0	1	2	3	4	5
y	7	5	9	19	35	57

Δy →	−2	4	10	16	22
$\Delta^2 y$ →	6	6	6	6	

Note that the value of c is the y-intercept.

This time, Δy is not constant between consecutive values of y. However, the difference between consecutive values of Δy (written $\Delta^2 y$) is 6.

Notice that 6 is twice as large as 3, the coefficient of x^2 in $y = 3x^2 - 5x + 7$. Does this mean $\Delta^2 y = 2a$ for any quadratic function of the form $y = ax^2 + bx + c$?

CAUTION

$\Delta^2 y$

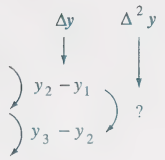
DOES NOT MEAN

$(\Delta y)^2$

Suppose y_1, y_2, and y_3 are consecutive y-values for consecutive x-values that differ by 1 (that is, $\Delta x = 1$).

In the table, $\Delta x = x_2 - x_1 = x_3 - x_2 = 1$.

x	y
x_1	$ax_1^2 + bx_1 + c$
x_2	$ax_2^2 + bx_2 + c$
x_3	$ax_3^2 + bx_3 + c$



$$\begin{aligned}
\therefore\ y_3 - y_2 &= \left(ax_3^2 + bx_3 + c\right) - \left(ax_2^2 + bx_2 + c\right) \\
&= ax_3^2 - ax_2^2 + bx_3 - bx_2 + c - c \\
&= a\left(x_3^2 - x_2^2\right) + b\left(x_3 - x_2\right) \\
&= a\left(x_3 - x_2\right)\left(x_3 + x_2\right) + b\left(x_3 - x_2\right) \\
&= a(1)\left(x_3 + x_2\right) + b(1) \\
&= a\left(x_3 + x_2\right) + b
\end{aligned}$$

Similarly, $y_2 - y_1 = a\left(x_2 + x_1\right) + b$.

$$\begin{aligned}
\therefore\ \Delta^2 y &= \left[a\left(x_3 + x_2\right) + b\right] - \left[a\left(x_2 + x_1\right) + b\right] \\
&= ax_3 + ax_2 + b - ax_2 - ax_1 - b \\
&= a\left(x_3 - x_2\right) + a\left(x_2 - x_1\right) \\
&= a + a \\
&= 2a
\end{aligned}$$

1. For $f(x) = -4x^2 - x + 1$, what is the value of $\Delta^2 y$? Demonstrate this result using a table of values.

Compare your response with the one in the Appendix, Section 1: Follow-up Activities, Enrichment.

You will now find a quadratic equation from a table of values.

Example

Determine the quadratic function from its partial table of values.

x	0	1	2	3
y	4	3	12	31

Solution

Determine Δy and $\Delta^2 y$.

x	0	1	2	3
y	4	3	12	31
Δy		−1	9	19
$\Delta^2 y$			10	10

Because $\Delta^2 y = 10$,

$$2a = 10$$
$$a = 5$$

Also, because $(0, 4)$ satisfies the equation, $c = 4$.

Therefore, $y = 5x^2 + bx + 4$.

To find the value of b, you can use any point other than $(0, 4)$.

Substitute $(1, 3)$ into the equation to find b.

$$y = 5x^2 + bx + 4$$
$$3 = 5(1)^2 + b(1) + 4$$
$$3 = 5 + b + 4$$
$$3 = b + 9$$
$$b = -6$$

Therefore, the quadratic function is $y = 5x^2 - 6x + 4$.

2. Determine the quadratic function from its partial table of values.

x	0	1	2
y	10	3	12

Compare your response with the one in the Appendix, Section 1: Follow-up Activities, Enrichment.

You will now use this method to derive the formula for the distance, d, an object falls after t seconds.

Example

Two students are working on an experiment where they must measure the distance, d, (in metres) an object falls from rest after 1s, 2s, and so on. Here are their results:

t	0	1	2	3	4	5
d	0	4.9	19.6	44.1	78.4	122.5

What function can be used to model this situation?

Solution

Calculate Δd and $\Delta^2 d$ to see if free fall can be modelled by either a linear or quadratic function.

t	0	1	2	3	4	5
d	0	4.9	19.6	44.1	78.4	122.5

Because $\Delta^2 d$ is constant, a quadratic function is an appropriate model. Let the function be $d = at^2 + bt + c$.

Because $\Delta^2 d = 9.8$,

$$2a = 9.8$$
$$a = 4.9$$

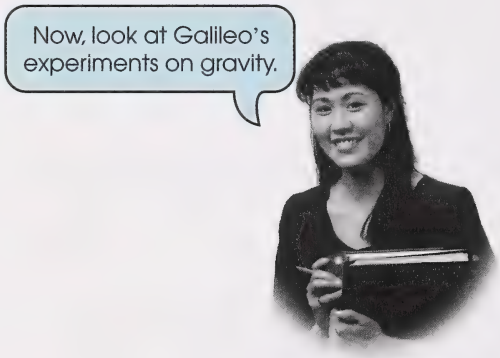
Also, because $(0, 0)$ lies on the graph, $c = 0$.

Now, the quadratic function is $d = 4.9t^2 + bt = 0$.

To find the value of b, substitute $(1, 4.9)$ into the equation.

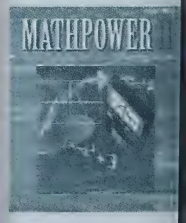
$$4.9 = 4.9(1)^2 + b(1)$$
$$4.9 = 4.9 + b$$
$$b = 0$$

Therefore, the quadratic function that models the distance, d, an object falls after t seconds is $d = 4.9t^2$.



3. Turn to "CONNECTING MATH AND HISTORY" on page 140 of *MATHPOWER™ 11* and read Investigation 1, "Falling Objects"; then answer questions 1 to 6.

Compare your responses with those in the Appendix, Section 1: Follow-up Activities, Enrichment.



Conclusion

In this section, you identified and graphed quadratic functions. You discovered that the graphs of all quadratic functions are parabolas. From the standard form of a quadratic function, you were able to determine the vertex, the domain and range, the axis of symmetry, the maximum or minimum value, and the intercepts of the graph. You then investigated the transformations of the graph of the basic quadratic function $y = x^2$ that produces $y = a(x - p)^2 + q$. If the quadratic function was not given in standard form, you transformed the function by completing the square. You then used your skills to model real-world situations and solve maximum and minimum problems.

NOAO

One of the applications of the quadratic function is modelling the orbits of comets as they approach the Sun, swing around the Sun, and return to deep space. The orbits are very close to being parabolic in shape, even for periodic comets. The preceding photograph is of Comet Halley as it appeared in 1986. Halley is a periodic comet, returning every 76 years to give people around the world a celestial show. Its most recent appearance was not as spectacular as it was in 1910, but there is always hope for a better show in 2062!

Assignment

Now, turn to Assignment Booklet 3A to complete the assignment for Section 1.

Section 2

Quadratic Equations

A well-trained diver executes a dive by gracefully twisting and turning almost effortlessly in the air, appearing to defy gravity, and then slicing into the water with little or no splash. It is easy to understand why the grace and skill of these athletes are so admired by spectators at competitions. Did you know that, in spite of the moves divers make after they leave the diving board, their centres of mass trace out parabolic trajectories?

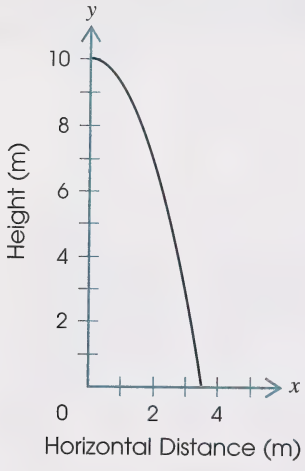
The motion of a diver's centre of mass can be modelled by a quadratic function. If this function is known, it can be used to form equations, which can be solved to further analyse a diver's performance.

In this section, you will solve quadratic equations and relate the solutions to the zeros of corresponding functions. You will also solve quadratic equations by graphing, factoring, taking square roots, and completing the square.

Activity 1: Solving Quadratic Equations by Graphing

The water that flows over the top of the falls in the photograph spills out one parabolic path after another as it cascades down the face of the cliff into the river below. If the quadratic function that models the flow were known, you could predict how far the water would travel horizontally before striking the pool below.

For example, consider the following parabola that models a waterfall 10 m high.



The water follows the curve described by the function $y = -0.8x^2 + 10$, where y is the vertical height (in metres) and x is the horizontal displacement (in metres).

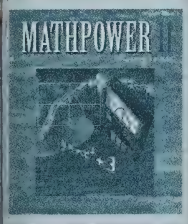
The total distance the water travels horizontally can be found by setting $y = 0$. The resulting equation, $0 = -0.8x^2 + 10$, is an example of a **quadratic equation**. A quadratic equation is an equation of the form $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$.

In this activity, you will solve quadratic equations by analysing the graphs of the related quadratic functions. For example, in the case of the waterfall, the value of x appears to be approximately 3.5 m.

Mathematical Process

- Communication
- [x] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- [x] Visualization

quadratic equation: an equation of the form $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$



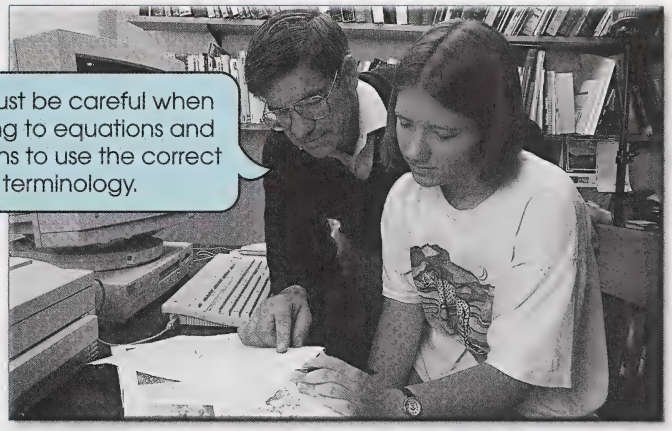
Turn to page 152 of *MATHPOWER™ 11* and read "Solving Quadratic Equations by Graphing."

1. Answer the following questions on page 152 of the textbook:

a. "Explore: Interpret the Graphs"
b. questions 1 to 4 of "Inquire"

Compare your responses with those in the Appendix, Section 2: Activity 1.

Mathematical Process
- [x] Communication
- [ ] Connection
- [ ] Estimation
- [ ] Mental Math
- [ ] Problem Solving
- [ ] Reasoning
- [ ] Technology
- [ ] Visualization



Quadratic Equation	Quadratic Function
$x^2 - x - 6 = 0$	$f(x) = x^2 - x - 6$
The roots are -2 and 3.	The zeros are -2 and 3.

root: a solution of an equation

A value that satisfies a quadratic equation is called a **root** of the equation. For example, 3 is a root of $x^2 - x - 6 = 0$.

LS	RS
$x^2 - x - 6$	0
$= 3^2 - 3 - 6$	
$= 9 - 3 - 6$	
$= 0$	

LS = RS

However, 3 is a zero of the function $f(x) = x^2 - x - 6$, since

$$f(3) = 3^2 - 3 - 6$$
$$= 0$$

Remember: A zero is a value of x for which $f(x) = 0$.

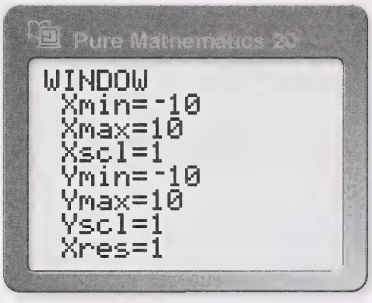
In the next example, you will solve a quadratic equation by graphing the related quadratic function on your graphing calculator and using its Zero feature.

Example

Solve $x^2 - x - 6 = 0$ by graphing the related quadratic function on your graphing calculator and using the calculator's Zero feature.

Solution

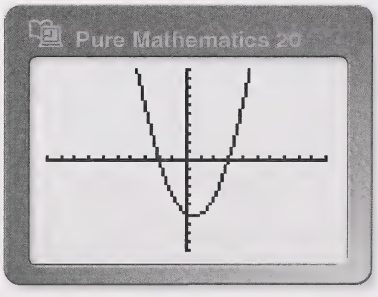
Before graphing, use the standard window settings so your calculator will display the same values as given in this example.



When working on different problems, you may have to change these settings to display the graph.

Graph the related function, $y = x^2 - x - 6$.

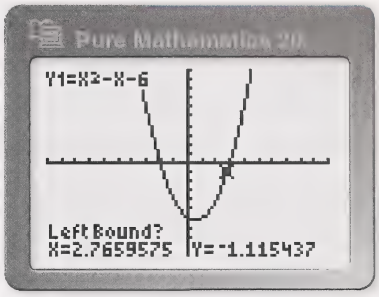
Y= X,T,Θ,n x^2 − X,T,Θ,n − 6 GRAPH



Mathematical Process

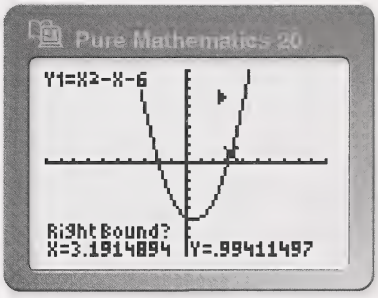
- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- [x] Technology
- Visualization

Use the Zero feature to determine the zeros of the function.



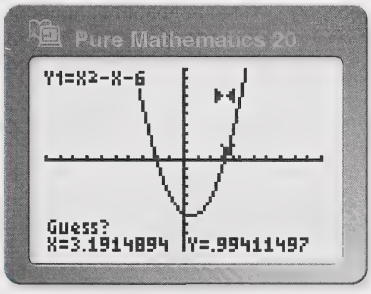
Press [2nd] [CALC] [2].

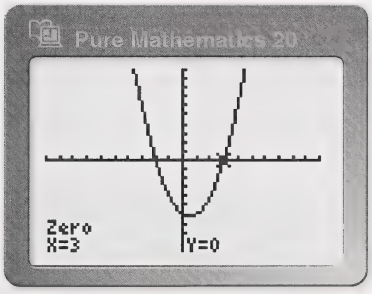
The cursor appears at $(0, -6)$, and the calculator prompts you to choose the left bound. Use the arrow keys to position the cursor just to the left of the x-intercept on the positive x-axis; then press [ENTER].



Now, the calculator prompts you to choose the right bound. Use the arrow keys to position the cursor just to the right of the x-intercept; then press [ENTER].

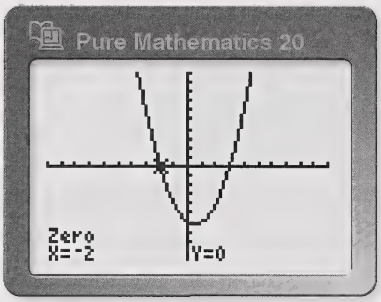
The calculator now prompts you to enter your guess. Tap the left arrow key once, positioning the cursor between the left and right bounds, and press [ENTER].





The calculator shows 3 as one of the zeros of the function.

Repeat this process to locate the negative x-intercept. Your display should look like the one on the right.



Therefore, the zeros of the function are -2 and 3.

The zeros are also the roots of the original quadratic equation, $x^2 - x - 6 = 0$. Check these roots by substituting $x = -2$ and $x = 3$ into the equation.

Check

For $x = -2$,

LS	RS
$x^2 - x - 6$	0
$= (-2)^2 - (-2) - 6$	
$= 4 + 2 - 6$	
$= 0$	
LS =	RS

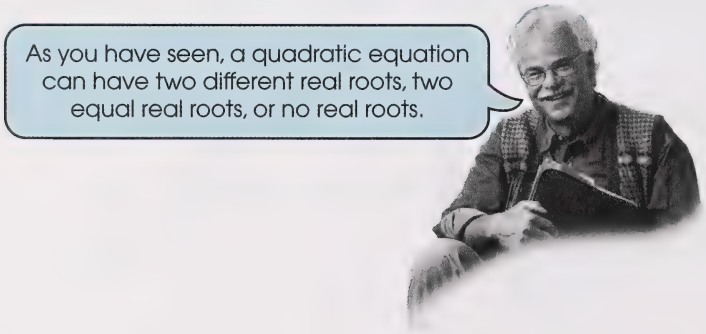
For $x = 3$,

LS	RS
$x^2 - x - 6$	0
$= 3^2 - 3 - 6$	
$= 9 - 3 - 6$	
$= 0$	
LS =	RS

Turn to pages 153 and 154 of *MATHPOWER™ 11* and work through Examples 1 through 3.

2. Answer questions 1, 7, 8, 22, and 30 of "Practice" on page 155 of the textbook.

Compare your responses with those in the Appendix, Section 2: Activity 1.



Example

For $x^2 + 6x + c = 0$, determine the values of c that would result in the equation having

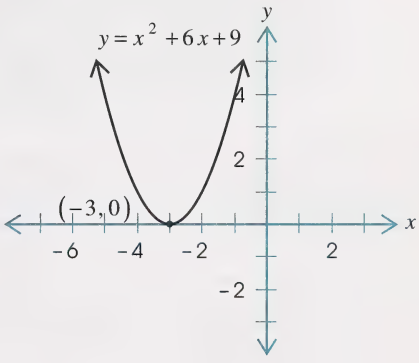
a. two real and equal roots
b. two real and different roots
c. no real roots

Solution

a. The quadratic equation $x^2 + 6x + c = 0$ has two real and equal roots if the graph of related quadratic function $y = x^2 + 6x + c$ touches the x-axis at one point. If the trinomial is a perfect square, the vertex will lie on the x-axis.

$$\therefore\ c = \left(\frac{6}{2}\right)^2$$
$$= 9$$

$$\therefore\ y = x^2 + 6x + 9$$
$$= (x+3)^2$$



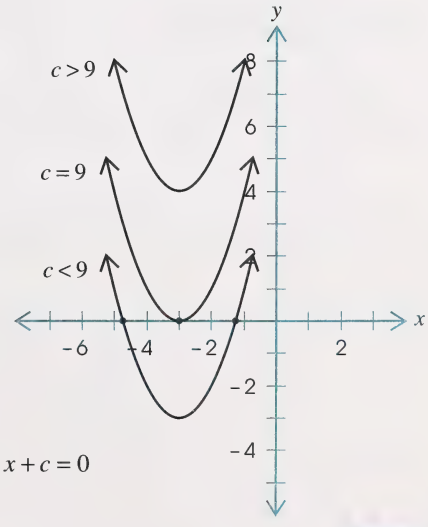
This graph opens upward from its vertex $(-3, 0)$.

Therefore, the roots of $x^2 + 6x + 9 = 0$ are both equal to 3 when $c = 9$.

b. Decreasing the value of c will move the graph of the quadratic function downward, thus making the graph cross the x-axis twice.

Therefore, if $c < 9$, the equation $x^2 + 6x + c = 0$ will have two real and different roots.

c. Increasing the value of c will move the graph of the quadratic function upward, thus not allowing the graph to cross the x-axis.



Therefore, if $c > 9$, the equation $x^2 + 6x + c = 0$ will have no real roots.

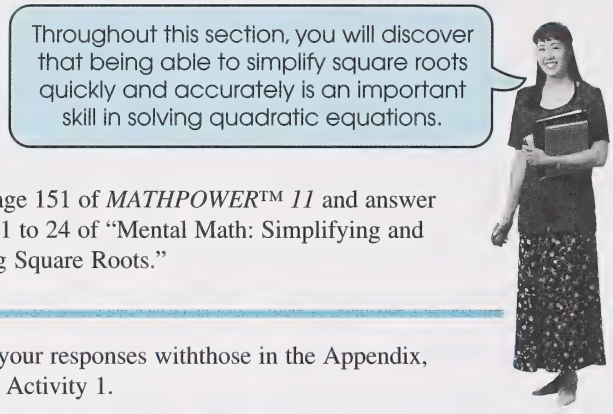
Turn to pages 154 and 155 of *MATHPOWER™ 11* and work through Examples 4 to 6.

3. Answer the following questions on pages 155 and 156 of the textbook:

a. question 15 of "Practice"
b. questions 41, 43, 45, 48, and 58 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 2: Activity 1.

Now Try This


Throughout this section, you will discover that being able to simplify square roots quickly and accurately is an important skill in solving quadratic equations.

4. Turn to page 151 of *MATHPOWER™ 11* and answer questions 1 to 24 of "Mental Math: Simplifying and Evaluating Square Roots."

Compare your responses withthose in the Appendix, Section 2, Activity 1.

Looking Back

In this activity, you solved quadratic equations for their roots by graphing the related quadratic functions and determining their zeros. The zeros of the function are the x-intercepts of its graph.

In your journal, list what you feel are the advantages and disadvantages of determining roots of equations from the graphs of the related functions.

Mathematical Process
- Communication
- Connection
- Estimation
- [X] Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

Mathematical Process
- [X] Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

Activity 2: Solving Quadratic Equations by Factoring

Mathematical Process

- Communication
- [X] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

Have you noticed how cheerful and welcoming a room can appear, even in midwinter, when sunlight streams in through a window? The amount of light a window admits is a function of its area. The dining-room window in the photograph is 1 m longer than it is wide. Its area is 6 m^2. What are the window's length and width?

To find the dimensions, identify the unknowns and set up an equation.

Let x be the width (in metres), and let $x+1$ be the length (in metres). Because the area of the window is 6 m^2, a quadratic equation you can use to solve this problem is

$$A = \text{length} \times \text{width}$$
$$6 = (x+1)x$$
$$6 = x^2 + x$$
$$x^2 + x - 6 = 0$$

This equation is an example of a **quadratic equation in standard form**.

$$ax^2 + bx + c = 0$$

When the equation is factored, the result is

$$(x+3)(x-2) = 0$$

Because the product of the factors is zero, either $x+3=0$ or $x-2=0$ or both are equal to 0.

From these equations, either $x = -3$ or $x = 2$. Because width is non-negative, the only admissible value of x is 2. Therefore, the width is 2 m and the length is $2 + 1 = 3$ m.

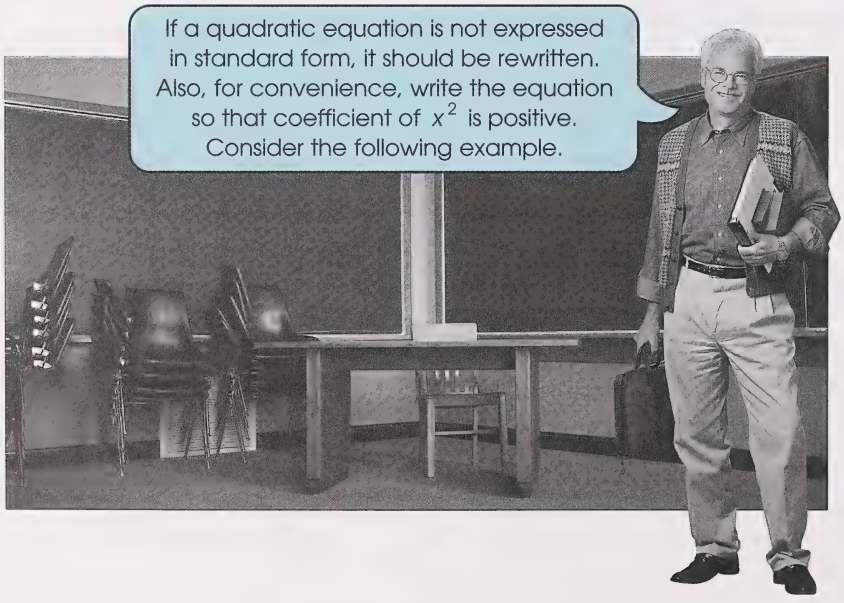
In this activity, you will solve quadratic equations by factoring. You will also solve problems that can be represented by quadratic equations.

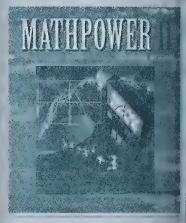
Turn to page 157 of *MATHPOWER™ 11* and read "Solving Quadratic Equations by Factoring."

1. Answer the following questions on page 157 of the textbook:

a. questions a. to c. of "Explore: Solve an Equation"
b. questions 1 to 4 of "Inquire"

Compare your responses with those in the Appendix, Section 2: Activity 2.


If a quadratic equation is not expressed in standard form, it should be rewritten. Also, for convenience, write the equation so that coefficient of x^2 is positive. Consider the following example.



Example

Solve $3x+7=-2x^2+4x+8$, and verify.

Solution

$$3x+7=-2x^2+4x+8$$

$2x^2-x-1=0$ ← Write the quadratic in standard form.

$(2x+1)(x-1)=0$ ← Factor.

$\therefore\ 2x+1=0$ or $x-1=0$ ← Use the zero product property.

$2x=-1$ $\qquad$ $x=1$

$x=-\frac{1}{2}$

Check

For $x=-\frac{1}{2}$,

LS	RS
$3x+7$	$-2x^2+4x+8$
$=3\left(-\frac{1}{2}\right)+7$	$=-2\left(-\frac{1}{2}\right)^2+4\left(-\frac{1}{2}\right)+8$
$=-\frac{3}{2}+\frac{14}{2}$	$=-2\left(\frac{1}{4}\right)-2+8$
$=\frac{11}{2}$	$=-\frac{1}{2}-\frac{4}{2}+\frac{16}{2}$
	$=\frac{11}{2}$

LS = RS

For $x=1$,

LS	RS
$3x+7$	$-2x^2+4x+8$
$=3(1)+7$	$=-2(1)^2+4(1)+8$
$=10$	$=10$

LS = RS

The roots are $-\frac{1}{2}$ and 1.

Turn to pages 158 and 159 of *MATHPOWER™ 11* and work through Examples 1 to 3.

2. Answer questions 6, 7, 12, 13, 25, 26, 32, 37, 44, 65, and 66 of "Practice" on pages 160 and 161.

Compare your responses with those in the Appendix, Section 2: Activity 2.

You must be careful when you are asked to solve a rational or fractional equation with variables in the denominator.

Example

Solve $\frac{1}{x-1}+\frac{1}{x+1}=\frac{x}{x-1}$.

Solution

For this equation, $x \neq \pm 1$. If x is replaced by 1, the fractions $\frac{1}{x-1}$ and $\frac{x}{x-1}$ are undefined. Similarly, if x is replaced by -1, the fraction $\frac{1}{x+1}$ is undefined. Therefore, if either $+1$ or -1 appears among the solutions, it must be discarded as an inadmissible root.

To solve the original equation, multiply by the lowest common denominator, $(x-1)(x+1)$.

$$\frac{1}{x-1}+\frac{1}{x+1}=\frac{x}{x-1}$$

$1(x+1)+1(x-1)=x(x+1)$ ← Multiply both sides by $(x-1)(x+1)$.

$x+1+x-1=x^2+x$ ← Expand.

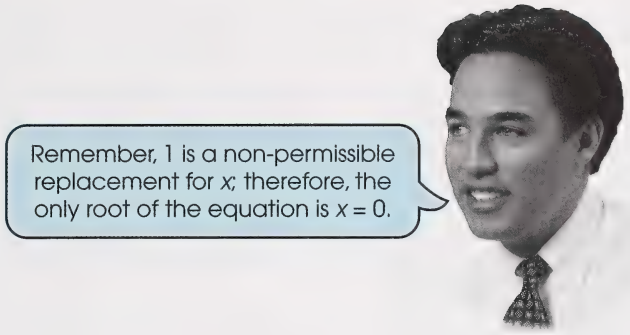
$2x=x^2+x$ ← Simplify.

$x^2-x=0$ ← Express in standard form.

$x(x-1)=0$ ← Factor.

$\therefore x=0$ or $x-1=0$ ← Use the zero property.

$x=1$



3. Verify that $x = 0$ is a root of the equation $\frac{1}{x-1} + \frac{1}{x+1} = \frac{x}{x-1}$.

Compare your response with the one in the Appendix, Section 2: Activity 2.

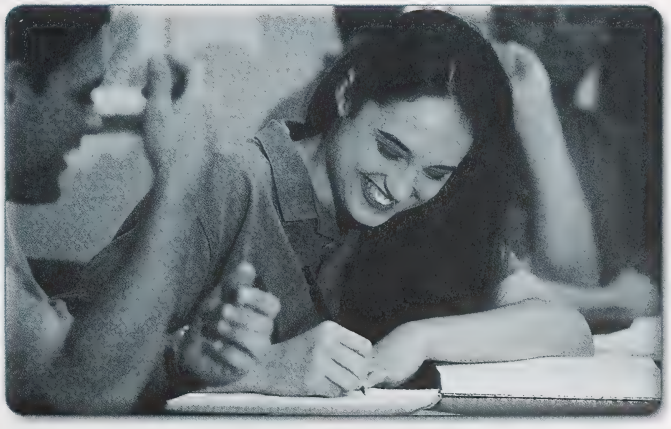
Turn to pages 159 to 162 of *MATHPOWER™ 11* and work through Examples 4 to 6.

4. Answer the following questions on pages 161 and 162 of the textbook:

a. questions 50, 53, 59, 60, and 62 of "Practice"

b. questions 69, 73, 75, 76, 81, 82, 86, 89, and 90 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 2: Activity 2.



Now Try This

Once you have solved several quadratic equations by factoring and your skills grow, you become more expert. Soon, the procedures seem routine. The problems and their solutions are part of a familiar pattern: a pattern that becomes part of your repertoire of equation-solving skills.

However, an unfamiliar pattern may take time to decipher. Only when you unravel it do you wonder why it took so long to recognize!

5. Turn to page 162 of *MATHPOWER™ 11* and answer questions 1 and 2 of "PATTERN POWER."

Compare your responses with those in the Appendix, Section 2: Activity 2.

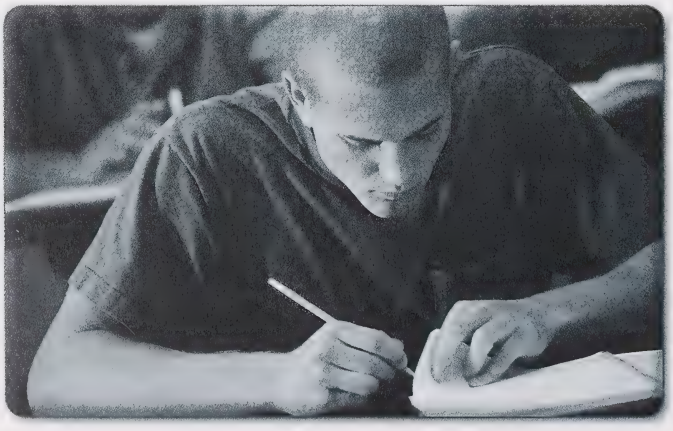
Looking Back

In this activity, you solved quadratic equations by first writing the equations in standard form, then factoring the quadratic, and then applying the zero property of a product.

In your journal, write quadratic equations that can be solved by factoring and that have

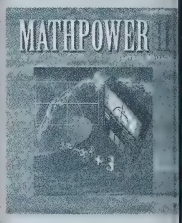
- two equal negative roots
- two different roots, where one is a negative fraction and the other is a negative integer

Give these equations to another student taking Pure Mathematics 20 and have him or her solve them.



Mathematical Process

- Communication
- Connection
- Estimation
- Mental Math
- [X] Problem Solving
- Reasoning
- Technology
- Visualization



Mathematical Process

- [X] Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

athematical
rocess
Communication
Connection
Estimation
Mental Math
Problem Solving
Reasoning
Technology
Visualization

Activity 3: The Square Root Principle

Have you seen footprints in the sand and wondered who left them? Was it an adult? a child? a man? a woman? The depth of the impression not only depends on the person's size or whether he or she was running or walking, it depends on the sand itself—whether it was wet or dry.

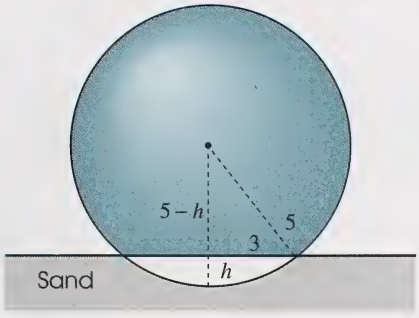
An impression in the sand can provide a lot of information about the person, animal, or object that created it or of the sand itself.

Consider a simple situation.

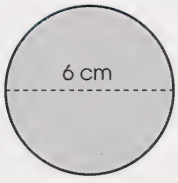
Example

Suppose a ball, 5 cm in radius, was dropped on the sand and left a circular impression 6 cm in diameter. What is the depth, h, of the impression?

Side View of Ball



Circular Impression
(as seen from above)



Solution

Let h be the depth (in centimetres) of the impression.

Use the Pythagorean Theorem and the sides of the right triangle to obtain the following quadratic equation:

$$(5-h)^2 + 3^2 = 5^2$$

Simplify.

$$(5-h)^2+9=25$$
$$(5-h)^2=16$$

Now, instead of writing the equation in standard form, take the square roots of both sides.

$$5-h=\pm\sqrt{16}$$
$$5-h=\pm 4$$

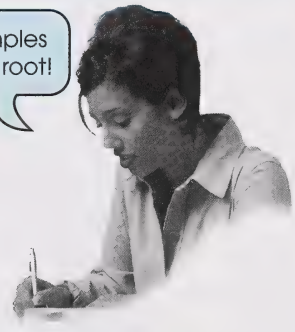
There are now two linear equations.

$5-h=4$	or	$5-h=-4$
$-h=-1$		$-h=-9$
$h=1$		$h=9$

Because the diameter of the ball is 10 cm and the diameter of the impression in the sand is only 6 cm, the depth of the impression must be 1 cm.

1. Solve $(5-h)^2=16$ by putting the equation in standard form and factoring the quadratic. Is the solution the same?

Compare your response with the one in the Appendix, Section 2: Activity 3.

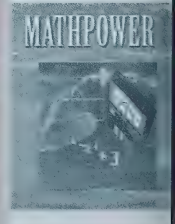


Turn to page 163 of *MATHPOWER™ 11* and read "Quadratic Equations—The Square Root Principle."

2. Answer the following questions on pages 163 and 164 of the textbook:

a. questions a. and b. of "Explore: Use the Diagram"
b. questions 1 to 7 of "Inquire"

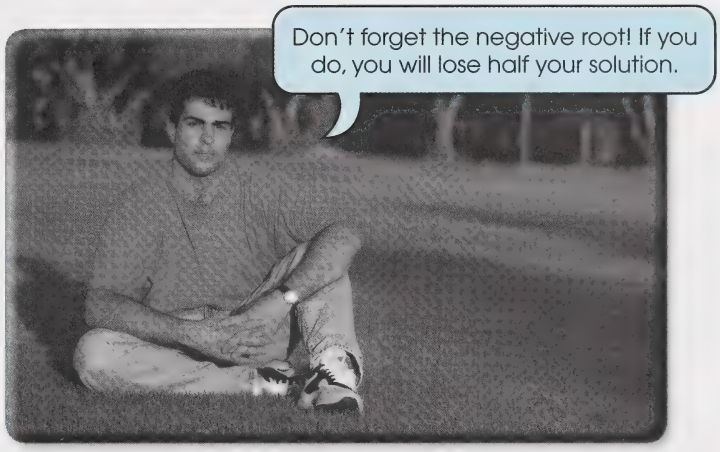
Compare your responses with those in the Appendix, Section 2: Activity 3.



As you have seen, if a quadratic equation has two distinct real solutions, they can be obtained from two factors of the quadratic or by extracting both square roots when the equation is reduced to the form

$x^2 = c$, where $c > 0$.

Here, the two solutions are $x = \sqrt{c}$ and $x = -\sqrt{c}$.



Turn to pages 164 and 165 of *MATHPOWER™ 11* and work through Examples 1 to 4. Look for two answers.

3. Answer the following questions on pages 165 to 167 of the textbook:

a. questions 4, 11, 18, 23, 40, 43, 51, 57, 65, and 69 of "Practice"

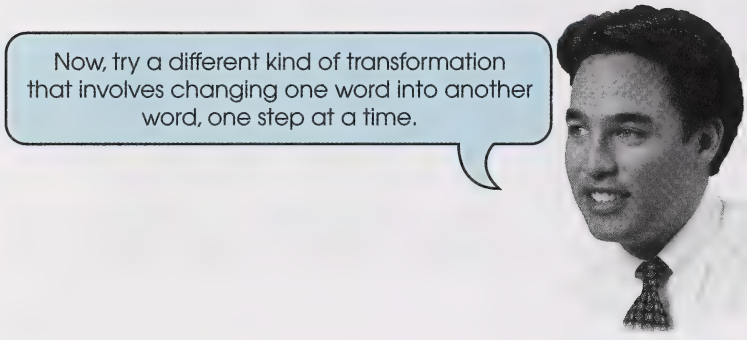
b. questions 74, 75, 76, 81, 85, 89.e., and 91 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 2: Activity 3.

Now Try This

Finding the solution to a quadratic equation by extracting square roots involves being able to transform a quadratic equation, one step at a time, from its given form into another equation of a different form. For example,

$$2x^2 - 3 = 47$$
$$2x^2 = 50 \quad \leftarrow \text{Add 3 to both sides.}$$
$$x^2 = 25 \quad \leftarrow \text{Divide both sides by 2.}$$
$$x = \pm\sqrt{25} \quad \leftarrow \text{Find the square root of both sides.}$$
$$= \pm 5 \quad \leftarrow \text{Simplify.}$$

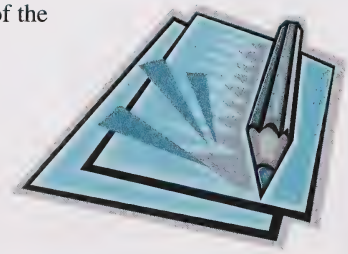


4. Turn to page 167 of *MATHPOWER™ 11* and answer "WORD POWER."

Compare your response with the one in the Appendix, Section 2: Activity 3.

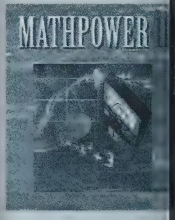
Looking Back

You should now be able to find the roots of equations of the form $ax^2 + c = 0$ by extracting square roots.



In your journal, describe how the processes of solving linear equations are similar to those of solving quadratic equations by taking square roots. Then describe how they are different.

Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- [X] Problem Solving
- Reasoning
- Technology
- Visualization



Mathematical Process
- [X] Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

Activity 4: Solving Quadratic Equations by Completing the Square

Mathematical Process
- Communication
- [x] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

Did you know that 4000 years ago the scribes of ancient Babylon recorded, on clay tablets, the details of the lives and commerce of the people who lived in the Tigris and Euphrates valleys? The fragment in the photograph is an example of the cuneiform (wedge-shaped characters) writing that appears on these tablets.

The level of mathematics these people attained was remarkable. Problems involving quadratic equations have been translated from these tablets. The method the scribes describe to solve these equations is a form of completing the square. By applying this method, they could solve equations of the form $ax^2 + bx = c$ for positive rational roots. Is it any wonder that this part of the world is called the "cradle of Western civilization"?

If you have access to the Internet you may wish to explore Babylonian mathematics further. One such site is as follows:

http://turnbull.dcs.st-and.ac.uk/history/HistTopics/Babylonian_and_Egyptian.html

In this activity, you will solve quadratic equations for both rational and irrational roots using the method of completing the square.

Turn to page 168 of *MATHPOWER™ 11* and read "Solving Quadratic Equations by Completing the Square."

1. Answer the following questions on pages 168 and 169 of the textbook:

a. questions a. and b. of "Explore: Use a Diagram"
b. questions 1 to 7 of "Inquire"

Compare your responses with those in the Appendix, Section 2: Activity 4.

Completing the square is based on the square root principle described in Section 2: Activity 2 of this module. You now have several methods you can use to solve quadratic equations: completing the square, taking square roots, factoring, and graphing. However, finding roots by graphing will only give you **approximate** values if the roots are irrational. Completing the square will give you **exact** values. Also, quadratic equations with irrational roots cannot be factored over the integers; so, you will have to resort to a method like completing the square to solve these equations.

Mathematical Process
- Communication
- [x] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

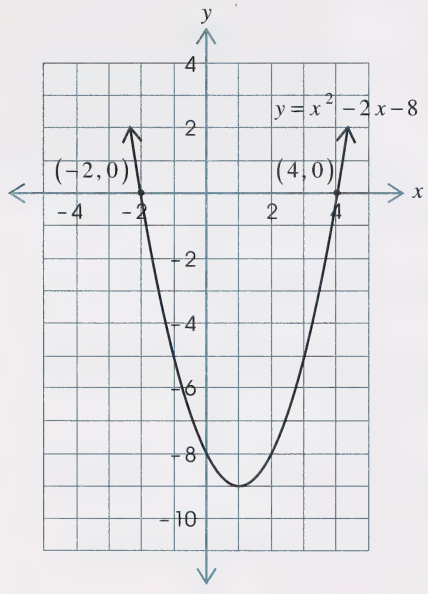
Example

Solve $x^2 - 2x - 8 = 0$ by

a. graphing
b. factoring
c. completing the square

Solution

a.



The x-intercepts of the graph are the zeros of the function $y = x^2 - 2x - 8$, which, in turn, are the roots of the equation $x^2 - 2x - 8 = 0$. The roots are 4 and -2.

b. $x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$ ← Factor the trinomial.

$\therefore\ x-4=0$ or $x+2=0$

$x = 4$ $\qquad$ $x = -2$

The roots are 4 and -2.

c. $x^2 - 2x - 8 = 0$

$x^2 - 2x = 8$ ← Add 8 to both sides.

$x^2 - 2x + 1 = 8 + 1$ ← Add the square of half the coefficient of x to both sides, $(-2 \div 2)^2 = 1$.

$x^2 - 2x + 1 = 9$

$(x-1)^2 = 9$ ← Write the left side as the square of a binomial.

$x - 1 = \pm\sqrt{9}$ ← Take the square root of both sides.

$x - 1 = \pm 3$

$\therefore\ x-1=3$ or $x-1=-3$

$x = 4$ $\qquad$ $x = -2$

The roots are 4 and -2.

Turn to pages 169 to 171 of *MATHPOWER™ 11* and work through Examples 1 to 4.

2. Answer the following questions on pages 172 and 173 of the textbook:

a. questions 4, 11, 15, 20, 24, 27, 33, 37, 41, 43, 49, and 52 of "Practice"

b. questions 57, 58, 60, 63, 67.a., 68, 72.a., and 72.d. of "Applications and Problem Solving."

Compare your responses with those in the Appendix, Section 2: Activity 4.

Now Try This

The method of completing the square is an example of an algorithm, a specific process for solving a particular problem. The pattern of steps can be repeated when you move from one problem to another. Some people argue that recognizing a pattern is fundamentally mathematics, whether the pattern involves numbers or not.

Try your pattern-recognition skills in the following problem.

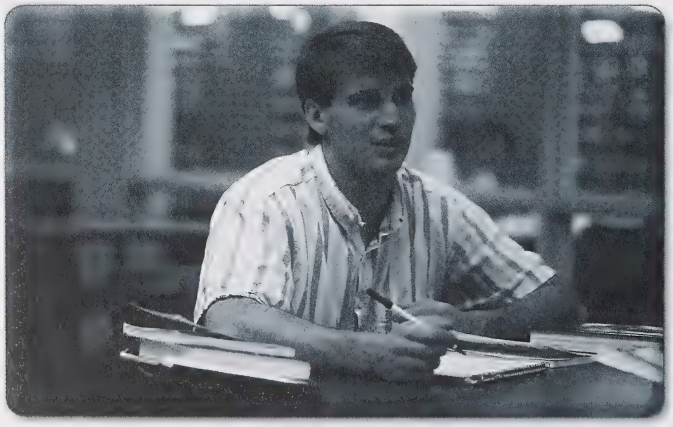
3. Turn to page 173 of *MATHPOWER™ 11* and answer "PATTERN POWER."

Compare your response with the one in the Appendix, Section 2: Activity 4.

Looking Back

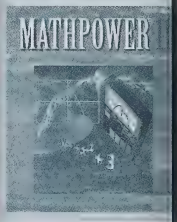
In this activity, you solved quadratic equations for both rational and irrational roots by completing the square.

The Babylonian scribes used only words and numbers when describing the solution to a quadratic equation; they did not use the modern symbols that you used in this activity. In your journal, pretend you are a Babylonian scribe and describe, step by step using only words and numbers, how to find the solution to $2x^2 - 13x = 1$ by completing the square.



Mathematical Process

- Communication
- Connection
- Estimation
- Mental Math
- [X] Problem Solving
- Reasoning
- Technology
- Visualization



Mathematical Process

- [X] Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

This section dealt with quadratic equations and their solution. You solved quadratic equations by graphing, factoring, taking square roots, and completing the square.

If you have access to the Internet, you will be able to review these basic skills by going to the following site:

http://www.sosmath.com/algebra/algebra.html

Example

Solve $x^2 - 2x - 2 = 0$, and check your answer.

Solution

Because you cannot factor the trinomial $x^2 - 2x - 2$ over the integers, solve the quadratic equation by completing the square.

$$x^2 - 2x - 2 = 0$$
$$x^2 - 2x = 2 \quad \text{Add 2 to both sides.}$$
$$x^2 - 2x + 1 = 2 + 1 \quad \text{Add the square of half the coefficient of } x \text{ to both sides, } (-2 \div 2)^2 = 1.$$
$$(x-1)^2 = 3 \quad \text{Write the left side as the square of a binomial.}$$
$$x - 1 = \pm\sqrt{3} \quad \text{Take the square roots of both sides.}$$

$$\therefore x - 1 = \sqrt{3} \quad \text{or} \quad x - 1 = -\sqrt{3}$$
$$x = 1 + \sqrt{3} \qquad x = 1 - \sqrt{3}$$

Check

For $x = 1 + \sqrt{3}$,

LS	RS
$x^2 - 2x - 2$	0
$= \left(1+\sqrt{3}\right)^2 - 2\left(1+\sqrt{3}\right) - 2$	
$= 1 + 2\sqrt{3} + 3 - 2 - 2\sqrt{3} - 2$	
$= 0$	

LS = RS

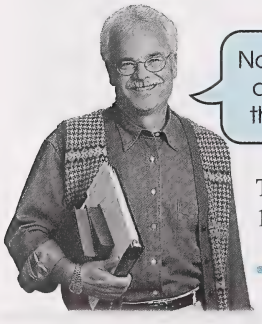
Check

For $x = 1 - \sqrt{3}$,

LS	RS
$x^2 - 2x - 2$	0
$= \left(1-\sqrt{3}\right)^2 - 2\left(1-\sqrt{3}\right) - 2$	
$= 1 - 2\sqrt{3} + 3 - 2 + 2\sqrt{3} - 2$	
$= 0$	

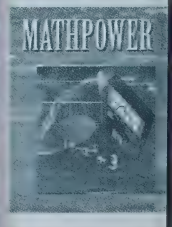
LS = RS

The solution is $1 + \sqrt{3}$ and $1 - \sqrt{3}$.



Turn to page 234 of *MATHPOWER™ 11* and answer questions 1, 5, 9, 12, 16, 18, and 20 of "Chapter Check."

Compare your responses with those in the Appendix, Section 2: Follow-up Activities, Extra Help.



Enrichment

In Activity 1 of this section, you solved for the roots of quadratic equations of the form $ax^2 + bx + c = 0$ by graphing the related quadratic functions $y = ax^2 + bx + c$ and determining the x-intercepts. You will now investigate how the graphs of the basic quadratic function, $y = x^2$, and a linear function can be used to determine the roots of a quadratic equation.

Example

Determine the roots of the $x^2 - 2x - 3 = 0$ from the points of intersection of the graphs of $y = x^2$ and a linear function.

Solution

Consider the system

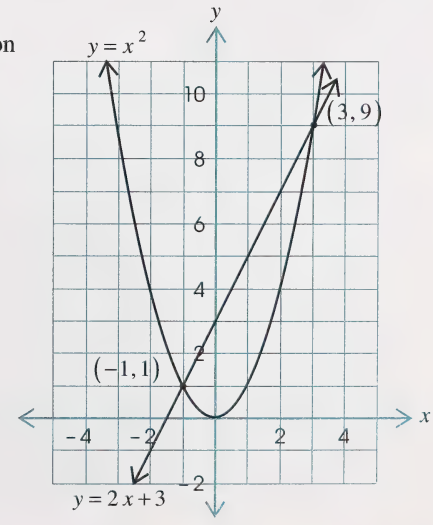
$$y = x^2 \quad (1)$$
$$y = 2x + 3 \quad (2)$$

If the graphs intersect, at the points of intersection, then $y = y$.

$$\therefore\ x^2 = 2x + 3$$
$$x^2 - 2x - 3 = 0$$

Therefore, the solution of the quadratic equation $x^2 - 2x - 3 = 0$ consists of the x-values of the points of intersection of the two graphs.

Graph the system.



The graphs appear to intersect at $(-1,1)$ and $(3,9)$.

The x-values of these points are -1 and 3; therefore, the roots of $x^2 - 2x - 3 = 0$ are -1 and 3.

Check

For $x = -1$,

LS	RS
$x^2 - 2x - 3$	0
$= (-1)^2 - 2(-1) - 3$	
$= 1 + 2 - 3$	
$= 0$	

LS = RS

Check

For $x = 3$,

LS	RS
$x^2 - 2x - 3$	0
$= 3^2 - 2(3) - 3$	
$= 9 - 6 - 3$	
$= 0$	

LS = RS

The roots are -1 and 3.

Now, you use this method to find the roots of the following:

1. $x^2 - 6x + 9 = 0$

2. $x^2 - 2x + 3 = 0$

Compare your responses with those in the Appendix, Section 2: Follow-up Activities, Enrichment.

Conclusion

In this section, you solved quadratic equations for their roots by graphing the related quadratic functions and identifying their x-intercepts (the zeros of these functions), by factoring the quadratic and using the zero-product principle, by extracting square roots, and by completing the square.

You discovered that the quadratic equation and quadratic function are related and that both can be used to model real-world problems, such as those involving free fall and the trajectories of projectiles.

Skilled divers perform artistic and complicated dives. Divers, tumblers, gymnasts, and acrobats twist and turn in the air, presenting the illusion that they are defying gravity. However, their motion conforms to the same physical laws as projectiles. The centre of masses of these performers, while in the air, trace out a parabolic path. The performers' motion can be modelled by quadratic functions and equations identical to those you investigated in this section.

Like many performers, your skills with quadratics will improve with practice!

Assignment

Now, turn to Assignment Booklet 3A to complete the assignment for Section 2.

Section 3

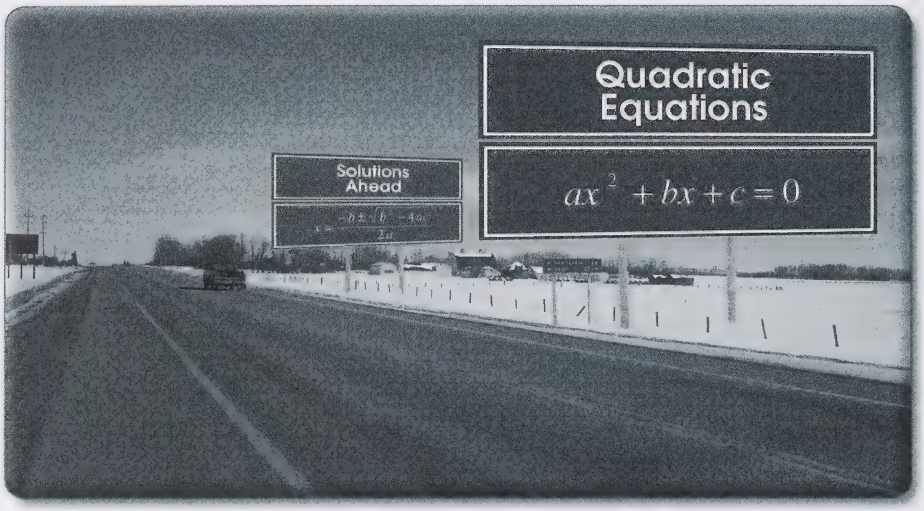
The Quadratic Formula

What factors determine how far you can jump? Is it just how fast you run? Does it depend on your take-off angle? What about your technique? Did you know that the Olympic record for the men's long jump is currently held by an American athlete by the name of Bob Beamon? In 1968, he leapt 8.90 m at the Summer Olympics held in Mexico City.

For a projectile, such as an arrow or a tossed ball, maximum range occurs when the initial angle to the horizontal is 45°. The trajectory of a projectile can be modelled by a quadratic function, and the range of the projectile can be determined by solving a quadratic equation.

In Activity 1 of this section, you will solve quadratic equations by applying a formula, called the quadratic formula. In Activity 2, you will discover that some of the roots you obtain are non-real and that the non-real roots can be combined using the basic operations of addition, subtraction, multiplication, and division. In Activity 3, you will explore the part of the quadratic formula that determines whether the solutions of a quadratic equation are real or non-real.

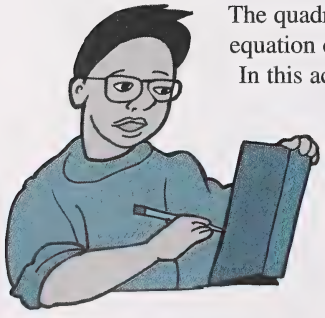
Activity 1: The Quadratic Formula



The quadratic formula represents the destination of a 4000-year journey begun by the ancient Babylonians, who were able to solve quadratic equations for rational roots by completing the square without modern notation. The Greek mathematicians then built on these beginnings. In 300 B.C., Euclid took a geometric approach to the solution of quadratic equations. By 100 A.D., Heron of Alexandria had mastered the quadratic.

If you have access to the Internet, you may wish to research the history of the quadratic equation and its solution.

http://turnbull.dcs.st-and.ac.uk/history/HistTopics/Quadratic_etc_equations.html



The quadratic formula describes how the roots of a quadratic equation can be obtained directly from the equation's coefficients. In this activity, you will derive this formula by completing the square on the general quadratic equation. Mathematical formulas such as this represent the refinement of a process and often conceal the underlying mathematics. Nevertheless, part of the power and elegance of this and other formulas is the notation that expresses the result concisely and unambiguously.

Mathematical Process

- [X] Communication
- [X] Connection
- [ ] Estimation
- [ ] Mental Math
- [ ] Problem Solving
- [ ] Reasoning
- [ ] Technology
- [ ] Visualization

You will now derive the quadratic formula.

Turn to page 174 of *MATHPOWER™ 11* and read "The Quadratic Formula."

1. Answer the following questions on pages 174 and 175 of the textbook:

a. "Explore: Describe the Steps"
b. questions 1 to 4 of "Inquire"

Compare your responses with those in the Appendix, Section 3: Activity 1.

There are a number of things that you should watch for when you apply the quadratic formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Example

Solve $-2x^2 = -4x - 3$ using the quadratic formula. Express your answer first in simplest radical form and correct to the nearest hundredth.

Solution

Before you can apply the quadratic formula, write the quadratic equation in standard form, $ax^2 + bx + c = 0$.

$$-2x^2 = -4x - 3$$
$$-2x^2 + 4x + 3 = 0$$

To avoid making errors in sign later on, it is better to write the equation so there is a positive value of a. You will obtain the correct answer if $a < 0$, but the algebra will be simpler if $a > 0$.

$$-2x^2 + 4x + 3 = 0$$
$$2x^2 - 4x - 3 = 0 \quad \longleftarrow \text{Multiply both sides by } -1.$$

$\therefore a = 2,\ b = -4$ and $c = -3$

Substitute these values into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

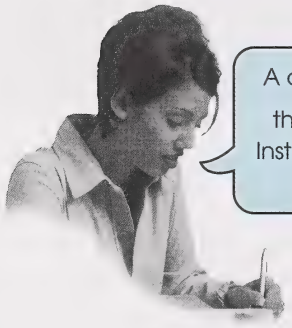
$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-3)}}{2(2)}$$

$$= \frac{4 \pm \sqrt{16 + 24}}{4}$$

$$= \frac{4 \pm \sqrt{40}}{4}$$

$$= \frac{4 \pm 2\sqrt{10}}{4}$$

Be careful! Because $b = -4$, use parentheses to avoid sign errors.



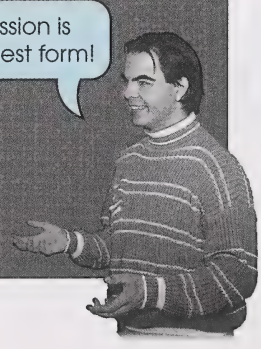
A common error at this point is to cancel the 4s and write $1 \pm 2\sqrt{10}$. Don't do this! Instead, there are two correct methods of simplifying this statement.

Method 1: Factoring the Numerator

$$x = \frac{4 \pm 2\sqrt{10}}{4}$$

$$= \frac{2(2 \pm \sqrt{10})}{4}$$

$$= \frac{2 \pm \sqrt{10}}{2}$$



The expression is now in simplest form!

Mathematical Process

- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- [X] Technology
- Visualization

Method 2: Separating, Reducing, and Recombining

$$x = \frac{4 \pm 2\sqrt{10}}{4}$$

$$= \frac{4}{4} \pm \frac{2\sqrt{10}}{4}$$

$$= \frac{2}{2} \pm \frac{\sqrt{10}}{2}$$ ← Reduce each part, but retain a common denominator.

$$= \frac{2 \pm \sqrt{10}}{2}$$ ← Recombine. You **cannot** cancel the 2s because you are adding and subtracting, not multiplying by 2; you **cannot** divide 2 into 10 because 10 is a radicand; and you **cannot** remove 2 as a common factor in the numerator because 10 is a radicand. You are done!

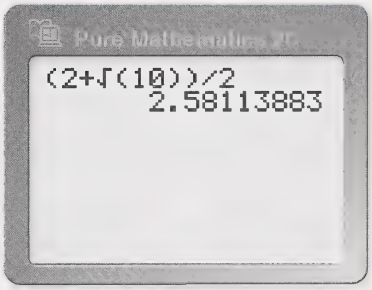
$$\therefore x = \frac{2+\sqrt{10}}{2} \quad \text{or} \quad x = \frac{2-\sqrt{10}}{2}$$

$$\doteq 2.58 \qquad \doteq -0.58$$

The roots are approximately 2.58 and -0.58.

To evaluate $\frac{2+\sqrt{10}}{2}$ using your graphing calculator, press

(2 + 2nd [√] 1 0) ÷ 2 ENTER



Now for more practice!

Turn to pages 175 to 178 of *MATHPOWER™ 11* and work through Examples 1 to 5.

2. Answer the following questions on pages 178 to 180 of the textbook:

a. questions 1, 4, 6, 9, 17, 23, 30, 34, 39, 46, 50, 54, 56, and 58 of "Practice"

b. questions 59.b., 63, 64, 66, 72, 77.a., 81, and 85.b. of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 3: Activity 1.

lathematical rocess

- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

Now Try This

One of the most famous sequences is the Fibonacci sequence:

1, 1, 2, 3, 5, 8, 13, 21, ...

Beginning with the third term, each term is the sum of the previous two terms.

This sequence is named after the twelfth-century Italian mathematician Leonardo of Pisa, better known as Leonardo Fibonacci (Fibonacci means "son of Bonacci").

The Fibonacci sequence is found extensively in nature. The number of petals of many flowers are numbers in this sequence. For example, the trillium has 3 petals, the prairie rose, 5 petals. The sequence is also found in the arrangement of leaves around a stem and the spirals of scales on a pineapple, seeds in a sunflower, and segments of a pinecone.

To find out more about this sequence, try the following investigation.

3. Turn to page 150 of *MATHPOWER™ 11* and answer the following:

a. questions 1 to 4 of Investigation 1, "The Sum of Consecutive Squares"
b. questions 1 to 3 of Investigation 2, "The Difference Between Alternate Squares"
c. questions 1 and 2 of Investigation 3, "Four Consecutive Fibonacci Numbers"

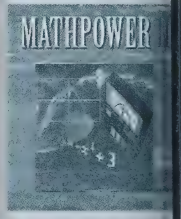
Compare your responses with those in the Appendix, Section 3: Activity 1.

Looking Back

In this activity, you solved quadratic equations and related problems using the quadratic formula.

In your journal, describe all the methods you now know for solving quadratic equations. List the advantages and disadvantages of each method, and indicate when you would use them. Illustrate with examples.

Mathematical Process
- Communication
- [X] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization



Mathematical Process
- Communication
- [X] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

Activity 2: Complex Numbers

Mathematical Process
- Communication
- [X] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

mplex number: the
m of a real number
d a pure
aginary number

At airshows today, modern aircraft perform feats that would have been impossible a few decades ago. They are marvellous to watch! These machines are a testimony to the remarkable skills of aeronautical engineers. One of the mathematical tools aircraft designers use is the complex number system.

In your previous work in mathematics, you encountered quadratic equations that have no real solution (such as $x^2 = -1$, where $\sqrt{-1}$ is undefined in the set of reals). In this activity, you will investigate the complex number system that contains the real numbers as a subset and supplies solutions to all quadratic equations.

Complex numbers have numerous applications. They are used in engineering, aerodynamics, fluid dynamics, electronics, the study of electromagnetism, elementary particle theory, general relativity, and the study of wave motion just to name a few. Chaos theory and fractals also involve complex numbers. In fact, some of the alien landscapes seen in science fiction movies are fractal landscapes.

The origins of complex numbers can be traced to the year 1545 when an Italian mathematician by the name of Girolamo Cardano used $\sqrt{-1}$ in the description of the roots of polynomial equations (of which quadratic and cubic equations are examples). In 1777, a Swiss mathematician, Leonard Euler, introduced the symbol i to represent $\sqrt{-1}$ and the modern representation for complex numbers $a + bi$ (which you will be studying in this activity). One of the world's greatest mathematicians, Carl Friedrich Gauss (1777–1855), placed complex numbers in a more concrete footing: identifying them by points in the plane.

If you have access to the Internet, you can explore the introduction of complex numbers in mathematics.

http://www.math.toronto.edu/mathnet/questionCorner/complexorigin.html

Turn to page 181 of *MATHPOWER™ 11* and read "Complex Numbers."

1. Answer the following questions on page 181 of the textbook:

a. questions a. to e. of "Explore: Use the Definitions"
b. questions 1 to 3 of "Inquire"

Compare your responses with those in the Appendix, Section 3: Activity 2.

You have just discovered that a **pure imaginary number** is of the form bi, where b is a non-zero real number and $i = \sqrt{-1}$. Examples of pure imaginary numbers are $-6i$, $2i$, and $i\sqrt{3}$.

pure imaginary number: a number of the form bi, where $i = \sqrt{-1}$ and b is a non-zero, real number

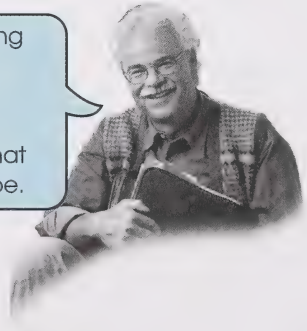
Recall that every positive real number has two square roots. For example, $\pm\sqrt{16} = \pm 4$. Similarly, every negative real number has two imaginary square roots. For example,

$$\begin{aligned}(6i)^2 &= 36i^2 \\ &= 36(-1) \\ &= -36\end{aligned}$$

$\therefore \sqrt{-36} = 6i$

This means $-\sqrt{-36} = -6i$.

You must be extremely careful when multiplying roots of negative numbers because the rule $\sqrt{x} \times \sqrt{y} = \sqrt{xy}$ should be applied to only non-negative values of x and y. This is an agreement adopted by mathematicians so that there is only one answer to products of this type.



Example

Simplify the following:

a. $\sqrt{4}\times\sqrt{9}$ **b.** $\sqrt{-4}\times\sqrt{-9}$ **c.** $\sqrt{-2}\times\sqrt{-6}$

Solution

a. $\sqrt{4}\times\sqrt{9}=\sqrt{4\times9}$
$=\sqrt{36}$
$=6$

You **can** do this, because $4>0$ and $9>0$.

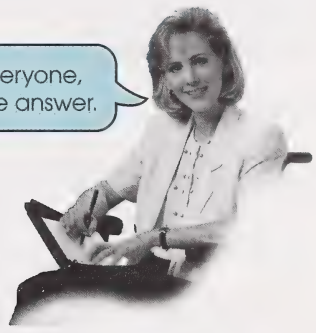
This answer makes sense, because $\sqrt{4}\times\sqrt{9}=2\times3=6$.

b. You **cannot** use the same method as in a. because $-4<0$ and $-9<0$. You must approach the problem this way:

$$\sqrt{-4}\times\sqrt{-9}=2i(3i)$$
$$=6i^2$$
$$=6(-1)$$
$$=-6$$

If you used the same method as in part a., you would get the following answer:

$$\sqrt{-4}\times\sqrt{-9}=\sqrt{-4\times(-9)}$$
$$=\sqrt{36}$$
$$=6 \leftarrow \text{WRONG!}$$



c. $\sqrt{-2}\times\sqrt{-6}=i\sqrt{2}\left(i\sqrt{6}\right)$
$=i^2\sqrt{12}$
$=-1\left(2\sqrt{3}\right)$
$=-2\sqrt{3}$

Remember: *Because -2 and -6 are negative, by agreement, you cannot write $\sqrt{(-2)(-6)}$.*

Now that you can handle pure imaginary numbers, you will explore complex numbers. A complex number is the sum of a real number, a, and a pure imaginary number, bi. This means that a complex number is of the form $a+bi$, where a and b are real numbers (for example, $3+2i$).

Turn to page 182 of *MATHPOWER™ 11* and read from the top of the page to the red line on page 183, working through Examples 1 to 3.

2. Answer the following questions on pages 185 and 186 of the textbook:

a. questions 1, 4, 7, 10, 16, 19 to 22, 29, 32, 34, 36, 44, 46, 48, 51, 53, and 54 of "Practice"

b. questions 88, 89.a., 91.a., and 91.c. of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 3: Activity 2.

If you have access to the Internet, you may wish to review the definitions of complex numbers and the operations of addition, subtraction, and multiplication. There are multiple sites that review these basics. One you might try is as follows:

http://bass.gmu.edu/~mazel/complex/complex.htm

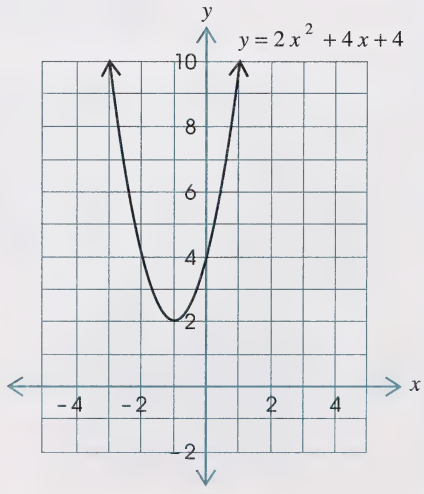
The reason you are studying complex numbers in this module is to complete the discussion of quadratic equations and their roots. You will recall that a quadratic equation may have two distinct real roots, two equal real roots, or non-real roots. You will now investigate the non-real roots.

Example

Solve $2x^2+4x+4=0$.

Solution

If you were to try to solve this equation graphically, you would begin by graphing the related quadratic function $y=2x^2+4x+4$.



The graph lies entirely above the x-axis. The function has no real zeros; the equation has no real roots.

Complex roots can be obtained using the quadratic formula.

$$2x^2+4x+4=0$$
$$x^2+2x+2=0 \quad \longleftarrow \text{Divide both sides by 2.}$$

$\therefore\ a=1,\ b=2,$ and $c=2$

Substitute these values into the quadratic formula.

$$\begin{aligned} x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a} \\ &= \frac{-2\pm\sqrt{2^2-4(1)(2)}}{2(1)} \\ &= \frac{-2\pm\sqrt{4-8}}{2} \\ &= \frac{-2\pm\sqrt{-4}}{2} \\ &= \frac{-2\pm 2i}{2} \\ &= -1\pm i \end{aligned}$$

The roots are the complex numbers $-1+i$ and $-1-i$.

Check

For $x=-1+i$,

LS	RS
$2x^2+4x+4$	0
$=2(-1+i)^2+4(-1+i)+4$	
$=2\left(1-2i+i^2\right)-4+4i+4$	
$=2(1-2i-1)+4i$	
$=2(-2i)+4i$	
$=-4i+4i$	
$=0$	

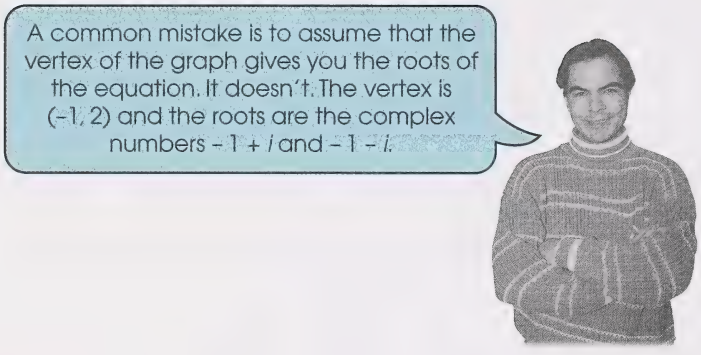
LS = RS

Check

For $x = -1 - i$,

LS	RS
$2x^2 + 4x + 4$	0
$= 2(-1-i)^2 + 4(-1-i) + 4$	
$= 2(1 + 2i + i^2) - 4 - 4i + 4$	
$= 2(1 + 2i - 1) - 4i$	
$= 2(2i) - 4i$	
$= 4i - 4i$	
$= 0$	

LS = RS


A common mistake is to assume that the vertex of the graph gives you the roots of the equation. It doesn't. The vertex is (-1, 2) and the roots are the complex numbers - 1 + i and - 1 - i.

Turn to page 183 of *MATHPOWER™ 11* and read from the red line to the red line near the bottom of page 185, working through Examples 4 to 7.

3. Answer the following questions on page 186 of the textbook:

a. questions 56, 61, 63, 68, 70, 75, 84, and 86 of "Practice"
b. questions 92.a., 92.b., 93.b., and 95 of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 3: Activity 2.

Now Try This

Your graphing calculator can be programmed to solve quadratic equations using the quadratic formula. The following program is for the *TI-83*. If you have a different calculator, you will have to modify the steps. Please consult your user's guide.

PROGRAM:QUADFORM
: ClrHome
: Disp "A="
: Input A
: Disp "B="
: Input B
: Disp "C="
: Input C
: $(-B+\sqrt{\ }(B^2-4AC))/(2A)\rightarrow D$
: $(-B-\sqrt{\ }(B^2-4AC))/(2A)\rightarrow E$
: Disp "ROOTS"
: Disp "X1="
: Disp D
: Disp "X2="
: Disp E

The keystrokes for each line follow.

To enter a new program, press PRGM and select the NEW menu. Your display should look like the one on the right.

Press ENTER. The screen now prompts you to name the program.

Program names can only be eight characters in length; therefore, name your program "QUADFORM" by pressing the following:

[Q] [U] [A] [D] [F] [O] [R] [M] ENTER

Now, enter the lines of the program.

PRGM [Select the I/O menu.] 8 ENTER

PRGM [Select the I/O menu.] 3 ALPHA ["] ALPHA [A] 2nd [TEST] 1 ALPHA ["] ENTER

PRGM [Select the I/O menu.] 1 ALPHA [A] ENTER

PRGM [Select the I/O menu.] 3 ALPHA ["] ALPHA [B] 2nd [TEST] 1 ALPHA ["] ENTER

PRGM [Select the I/O menu.] 1 ALPHA [B] ENTER

PRGM [Select the I/O menu.] 3 ALPHA ["] ALPHA [C] 2nd [TEST] 1 ALPHA ["] ENTER

PRGM [Select the I/O menu.] 1 ALPHA [C] ENTER

((–) ALPHA [B] + 2nd [$\sqrt{\ }$] ALPHA [B] x^2 − 4 ALPHA [A] ALPHA [C])) ÷ (2 ALPHA [A]) STO▶ ALPHA [D] ENTER

((–) ALPHA [B] − 2nd [$\sqrt{\ }$] ALPHA [B] x^2 − 4 ALPHA [A] ALPHA [C])) ÷ (2 ALPHA [A]) STO▶ ALPHA [E] ENTER

PRGM [Select the I/O menu.] 3 2nd [A-LOCK] ["] [R] [O] [O] [T] [S] ["] ALPHA ENTER

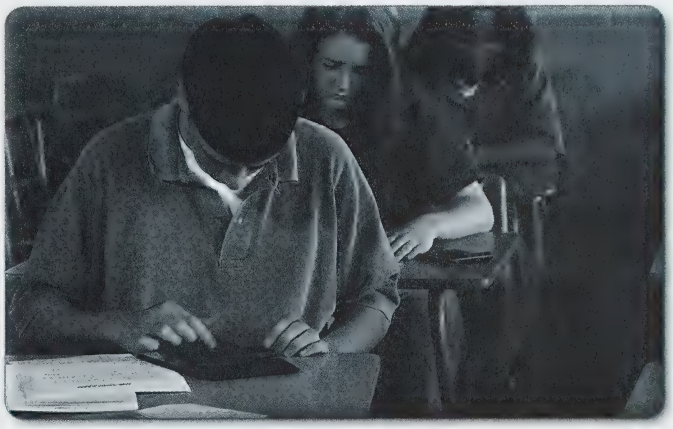
PRGM [Select the I/O menu.] 3 ALPHA ["] ALPHA [X] 1 2nd [TEST] 1 ALPHA ["] ENTER

PRGM [Select the I/O menu.] 3 ALPHA [D] ENTER

PRGM [Select the I/O menu.] 3 ALPHA ["] ALPHA [X] 2 2nd [TEST] 1 ALPHA ["] ENTER

PRGM [Select the I/O menu.] 3 ALPHA [E] ENTER

To run your program, press PRGM, select "QUADFORM," and press ENTER. You will then need to press ENTER a second time in order to enter the coefficients. As you are prompted to enter the coefficients, enter each specified value and press ENTER.



4. Solve each of the following equations using the QUADFORM program.

a. $x^2 - 3x - 4 = 0$
b. $4x^2 - 4x + 1 = 0$
c. $2x^2 - 3x - 7 = 0$

5. What happens when you try to solve $x^2 + 2x + 2 = 0$ using your program. Why?

Compare your responses with those in the Appendix, Section 3: Activity 2.

Looking Back

In this activity, you explored complex numbers as solutions to quadratic equations. You wrote complex numbers, found the square roots of negative reals, and performed the basic operations on complex numbers.

Occasionally, mathematicians are called mystics because they deal with irrational, transcendental, and imaginary numbers. Using your dictionary, write the mathematical definitions of these terms in your journal. Then write down the more common definitions of irrational, transcendental, and imaginary. Do you think the term "mystic" is an appropriate one for mathematicians? Write your explanation in your journal.

Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

Activity 3: The Discriminant

Do you like sour foods? When you see a picture of sliced lemons or limes, does your mouth water? Do you purse your lips? Can you feel your cheeks tingle? Could you bite into a slice of lime without wincing?

The human palate can discriminate among four tastes: sour, sweet, bitter, and salty. People become less sensitive to tastes as they age; young children have the most discriminating sense of taste. This may be part of the reason they reject some foods that adults enjoy.

The expression $b^2 - 4ac$, which is part of the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, is called the **discriminant**. The value of the discriminant allows you to discriminate among the possible kinds of solutions of quadratic equations:

- two real and different roots
- two real and equal roots
- two non-real roots

Turn to page 187 of *MATHPOWER™ 11* and read "The Discriminant."

1. Answer the following questions on page 187 of the textbook:

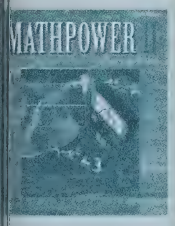
a. "Explore: Complete the Table"
b. questions 1 to 3 of "Inquire"

Compare your responses with those in the Appendix, Section 3: Activity 3.

Remember that the roots of a quadratic equation are the zeros of the related quadratic function. The zeros of the function are the x-intercepts of its graph.

Type of Solution for $ax^2 + bx + c = 0$	Number of x-intercepts of $y = ax^2 + bx + c$
2 real and distinct roots	2 distinct x-intercepts
2 real and equal roots	1 x-intercept
2 imaginary roots	0 x-intercepts

These relationships are illustrated in your textbook.



Turn to page 188 of *MATHPOWER™ 11* and read from the top of the page to the red line on page 189, working through Examples 1 to 3.

2. Answer the following questions on pages 189 and 190 of the textbook:

a. questions 1, 2, 3, 14, 15, 23, 26, 29, 30, 31, 35, 36, 42, and 43 of "Practice"
b. questions 45, 47, 49, 50, and 54.a. of "Applications and Problem Solving"

Compare your responses with those in the Appendix, Section 3: Activity 3.

Now Try This

In this activity, you described the roots of a quadratic equation without actually finding those roots. You simply used the formula for the discriminant. Did you know that you can also determine the sum of the roots and the product of the roots without actually knowing what the roots are? You can find the sum and product using formulas based on the coefficients of the quadratic equation!

Mathematical Process
- Communication
- [x] Connection
- Estimation
- Mental Math
- Problem Solving
- [x] Reasoning
- Technology
- Visualization

3. Turn to "INVESTIGATING MATH" on pages 192 and 193 of *MATHPOWER™ 11* and answer the following:

a. questions 1 to 5 and 6.a. and 6.c. of Investigation 1, "Exploring the Relationships"

b. questions 1 to 3 of Investigation 2, "Using Algebra"

c. questions 1.a. to 1.c. and 2.a. to 2.c. of Investigation 3, "Working Backward"

Compare your responses with those in the Appendix, Section 3: Activity 3.

Looking Back

In this activity, you explored the nature of the roots of a quadratic equation using the discriminant. You related the nature of the roots of the equation to the x-intercepts of the graph of its related function.

An Italian mathematician by the name of Girolamo Cardano (1501–1576) made famous the formula for calculating the roots of a cubic equation of the form $ax^3 + bx^2 + cx + d = 0$. Although this formula is known as Cardan's formula, Cardano did not derive the formula himself. He obtained it from Niccolo Tartaglia, a contemporary mathematician.

If you have access to the Internet, research the mathematicians Cardano and Tartaglia and Cardan's formula. Write the results of your research in your journal.

You may wish to begin your research with the following site:

http://viva.lib.virginia.edu/science/parshall/cardano.html

Mathematical Process

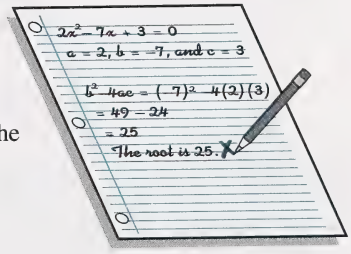
- [x] Communication
- [ ] Connection
- [ ] Estimation
- [ ] Mental Math
- [ ] Problem Solving
- [ ] Reasoning
- [ ] Technology
- [ ] Visualization

Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

A common mistake many students make is they confuse the value of the discriminant with the roots of the equation. Nevertheless, because the discriminant is part of the quadratic formula, it is possible to use the value of the discriminant in determining the nature of the roots and the roots themselves.



Example

For $2x^2 - 7x + 3 = 0$, determine the following:

a. the nature of the roots
b. the roots of the equation

Solution

a. To determine the nature of the roots, find the value of the discriminant.

$$2x^2 - 7x + 3 = 0$$

$\therefore a = 2,\ b = -7,$ and $c = 3$

Substitute these values into the formula for the discriminant.

$$\begin{aligned} b^2 - 4ac &= (-7)^2 - 4(2)(3) \\ &= 49 - 24 \\ &= 25 \end{aligned}$$

Because $b^2 - 4ac > 0$, the equation has two real and distinct roots.

b. To find the roots, use the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

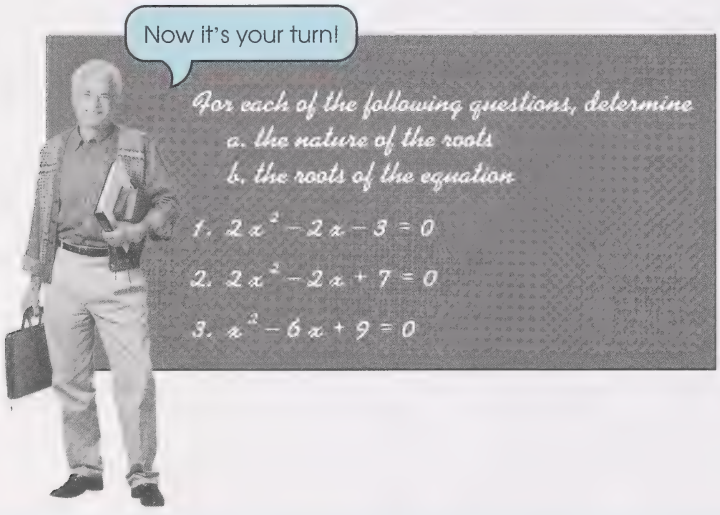
$$= \frac{-(-7) \pm \sqrt{25}}{2(2)} \quad \leftarrow \text{Substitute 25 for } b^2 - 4ac.$$

$$= \frac{7 \pm 5}{4}$$

$$\therefore x = \frac{7+5}{4} \quad \text{and} \quad x = \frac{7-5}{4}$$

$$= 3 \qquad\qquad = 0.5$$

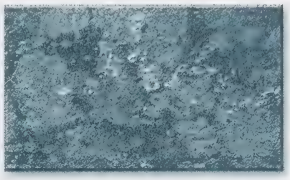
The roots are 3 and 0.5.


Now it's your turn!
For each of the following questions, determine
a. the nature of the roots
b. the roots of the equation
1. $2x^2 - 2x - 3 = 0$
2. $2x^2 - 2x + 7 = 0$
3. $x^2 - 6x + 9 = 0$

Compare your responses with those in the Appendix, Section 3: Follow-up Activities, Extra Help.

Enrichment

The ancient Greeks revered the golden rectangle, a rectangle whose length is ϕ times its width, where ϕ, the golden ratio, is approximately 1.618 033 989.



w

$\ell \doteq 1.618\ w$

The golden rectangle and the golden ratio appear extensively in the art and architecture of ancient Greece (as in the Parthenon, built in fifth-century B.C.). Today, you can find it not only in modern art and architecture, but also in the natural world as well.

If you have access to the Internet, you may wish to learn more about how the golden rectangle is used in art and mathematics by accessing the following websites:

http://www.zometool.com/deepzome/golden.html

http://forum.swarthmore.edu/dr.math/faq/faq.golden.ratio.html

Turn to "CONNECTING MATH AND ESTHETICS" on page 194 of *MATHPOWER™ 11* and read "The Golden Ratio."

Answer the following questions on pages 195 and 196 of the textbook:

1. questions 1 to 4 of Investigation 1, "Investigating ϕ"

2. questions 1 and 2 of Investigation 2, "Geometry and ϕ"

3. questions 1 to 3 of Investigation 3, "Fibonacci Numbers and ϕ"

4. question 1 of Investigation 4, "ϕ in Architecture, Design, and Nature"

Compare your responses with those in the Appendix, Section 3: Follow-up Activities, Enrichment.

Conclusion

In Activity 1 of this section, you solved quadratic equations for their roots by applying the quadratic formula. In Activity 2, you discovered that some of these roots are non-real and that the non-real roots belong to the set of complex numbers that contains the reals as a subset. You then found that complex numbers may be combined, as can real numbers, using the operations of addition, subtraction, and multiplication. Later, you solved the quadratic equations with non-real roots and wrote the solutions as complex numbers. In Activity 3, you investigated the nature of the roots of the quadratic equation using the discriminant.

Quadratic equations arise naturally when events are modelled by quadratic functions. When specific values of the independent variable, x, are required and values of the function, y, are known, you must solve a quadratic equation. For instance, if the parabolic path of the spray in the photograph is represented by a quadratic function $y = f(x)$, then set $y = 0$ (at ground level) and solve for x to find out how far the spray will reach. Even at poolside on a hot summer's day, quadratic equations are child's play!

Assignment

Now, turn to Assignment Booklet 3B to complete the assignment for Section 3.

Module Summary

In this module you explored a special class of functions and equations called quadratics. In Section 1, you graphed and described quadratic functions, expressed either in standard or general form. One of the critical processes you used in your analysis was completing the square. You completed the square to transform quadratic functions to standard form, to sketch and describe their graphs, and as a key element in determining maximum or minimum values of quadratic functions used to model real-world situations.

In Section 2, you applied several methods of solving quadratic equations. These methods included graphing related quadratic functions, factoring, extracting square roots, and completing the square.

In Section 3, you used the quadratic formula to determine roots. You also investigated the nature of the roots to quadratic equations and examined how to solve for non-real roots. You discovered that these roots are part of the set of complex numbers, realizing that real numbers belong to this larger set of numbers, too!

The burst of fireworks lighting the night sky over the suspension bridge in the harbour is spectacular. Canada Day and New Year's Eve displays draw large and appreciative crowds. Do you remember a particularly impressive display? The shower of sparks from a burst trace out parabolic trajectories as they slowly fade from view. Trajectories, orbits of comets, parabolic mirrors and antennas, and extreme-value problems were but a few of the applications of quadratic equations and functions you investigated in this module. Throughout your school career you will encounter additional uses of the techniques you studied in this module!

Final Module Assignment

Now, turn to Assignment Booklet 3B to complete the Final Module Assignment.

Glossary
Suggested Answers

Glossary

axis of symmetry: a line that divides a curve into halves, reflecting each half into the other

completing the square: the process of transforming a quadratic function from general form to standard form

complex number: a number of the form $a+bi$, where $i=\sqrt{-1}$

discriminant: a part of the quadratic formula, given by b^2-4ac

general form of a quadratic function: a quadratic function written in the form $y=ax^2+bx+c$, where a, b, and c are real numbers and $a \neq 0$

parabola: a U-shaped curve obtained by graphing a quadratic function

paraboloid of revolution: a three-dimensional figure formed by rotating a parabola about its axis of symmetry

pure imaginary number: a number of the form bi, where $i=\sqrt{-1}$ and b is a non-zero real number

quadratic equation: an equation of the form $ax^2+bx+c=0$, where a, b, and c are real numbers and $a \neq 0$

quadratic formula: the formula for determining the roots of a quadratic equation from its coefficients

quadratic function: a function that may be written in the form $y=ax^2+bx+c$, where a, b, and c are real numbers and $a \neq 0$

root: a solution of an equation

standard form of a quadratic equation: a quadratic equation written in the form $ax^2+bx+c=0$, where a, b, and c are real numbers and $a \neq 0$

standard form of a quadratic function: $y=a(x-p)^2+q$, where a, p, and q are real numbers and $a \neq 0$

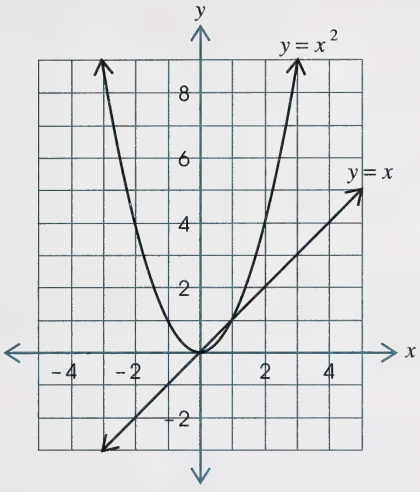
vertex: the point of intersection of a parabola and its axis of symmetry

zero: the value of the independent variable of a function for which the function equals zero

Suggested Answers

Section 1: Activity 1

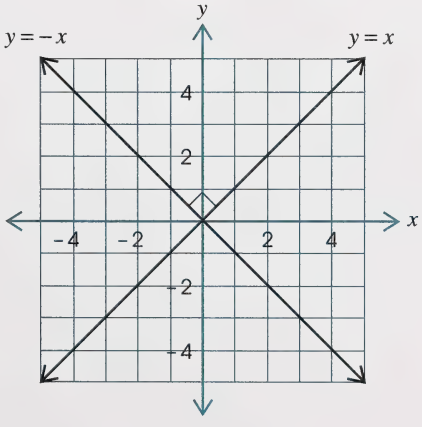
1. a. Textbook question "Explore: Compare the Graphs," p. 104



b. Textbook questions 1 to 5 of "Inquire," p. 104

1. The graph of $y = x^2$ is symmetrical with respect to the y-axis.

2. The graph of $y = x$ has an infinite number of lines of symmetry. It is symmetrical with respect to itself and to any line that crosses it at a right angle. The line $y = -x$, for example, is a line of symmetry of $y = x$.



Section 1: Activity 1 (continued)

3. **a.** The graph of $y = x$ does not have a minimum or maximum value of x. The line can be extended infinitely to the left and to the right. Any real value of x may be substituted into the equation.

b. The graph of $y = x$ does not have a minimum or maximum value of y. The line can be extended infinitely upward and downward. Any real value of y can be obtained from the equation.

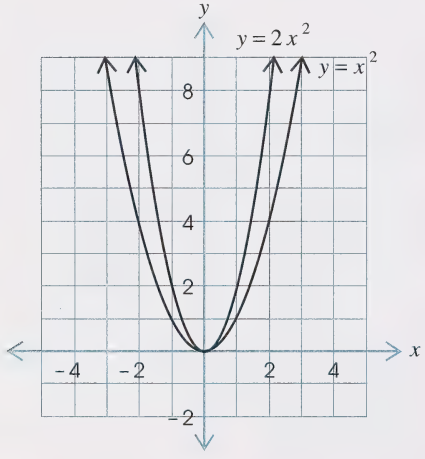
c. The domain is $x \in R$, and the range is $y \in R$.

4. **a.** The graph of $y = x^2$ does not have a minimum or maximum value of x. Any real number may be substituted for x.

b. The graph of $y = x^2$ has a minimum value of y, but no maximum value. Because y is obtained by squaring any real number, y is non-negative; its minimum value is 0. The lowest point on the graph of $y = x^2$ occurs at $(0, 0)$.

c. The domain is $x \in R$, and the range is $y \geq 0$.

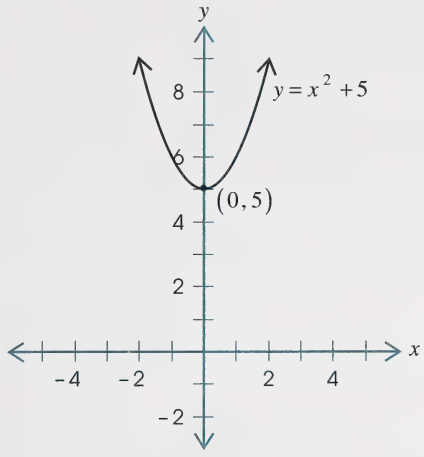
5.



The graph of $y = 2x^2$ is narrower than the graph of $y = x^2$.

2. Textbook questions 1 to 4, 13, and 14 of "Practice," p. 109

1.



a. The graph opens upward.

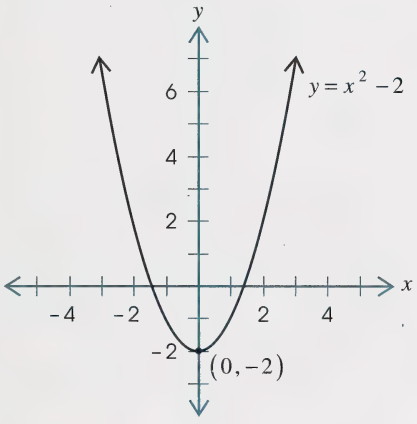
b. The vertex is $(0, 5)$.

c. The axis of symmetry is $x = 0$.

d. The domain is $x \in R$, and the range is $y \geq 5$.

e. The minimum value is 5.

2.



a. The graph opens upward.

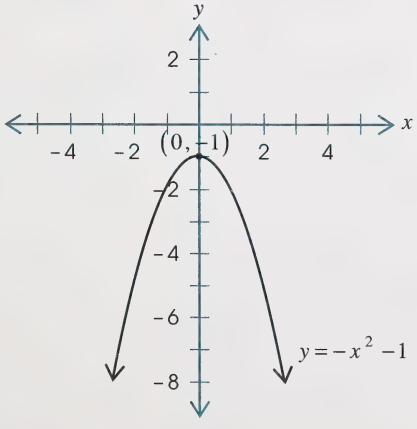
b. The vertex is $(0, -2)$.

c. The axis of symmetry is $x = 0$.

d. The domain is $x \in R$, and the range is $y \geq -2$.

e. The minimum value is -2.

3.



a. The graph opens downward.

b. The vertex is $(0, -1)$.

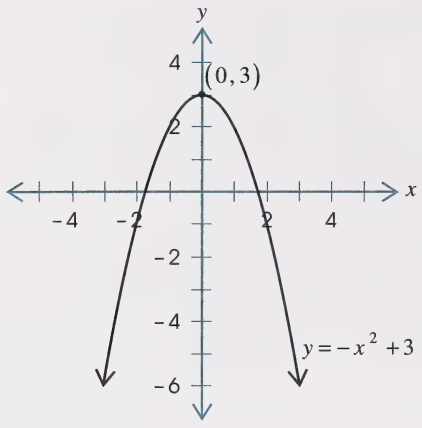
c. The axis of symmetry is $x = 0$.

d. The domain is $x \in R$, and the range is $y \leq -1$.

e. The maximum value is -1.

Section 1: Activity 1 (continued)

4.



a. The graph opens downward.

b. The vertex is $(0, 3)$.

c. The axis of symmetry is $x = 0$.

d. The domain is $x \in R$, and the range is $y \leq 3$.

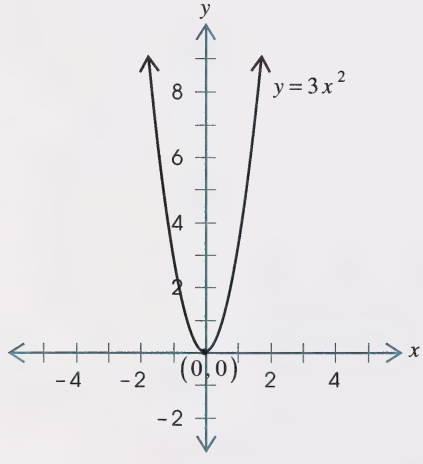
e. The maximum value is 3.

13. The graph of $y = x^2 - 4$ is the graph of $y = x^2$ translated 4 units downward.

14. The graph of $y = -x^2 + 5$ is the graph of $y = -x^2$ translated 5 units upward.

3. Textbook questions 5, 6, 8, 15, 16, and 18 of "Practice," p. 109

5.



a. The graph opens upward.

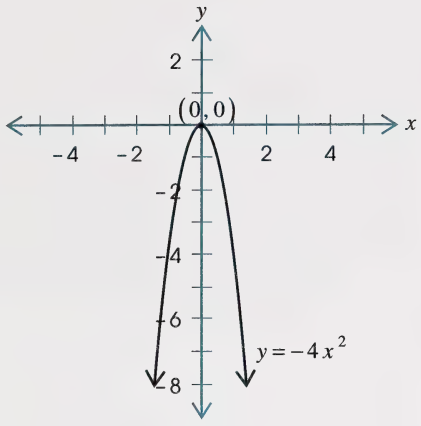
b. The vertex is $(0, 0)$.

c. The axis of symmetry is $x = 0$.

d. The domain is $x \in R$, and the range is $y \geq 0$.

e. The minimum value is 0.

6.



a. The graph opens downward.

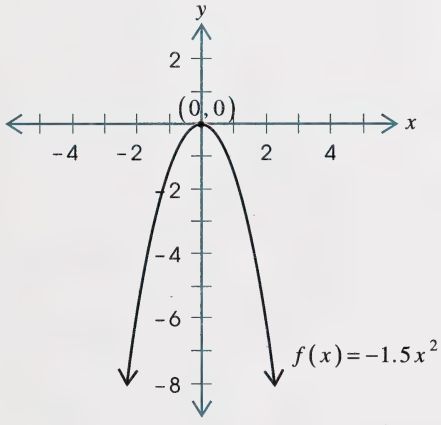
b. The vertex is $(0, 0)$.

c. The axis of symmetry is $x = 0$.

d. The domain is $x \in R$, and the range is $y \leq 0$.

e. The maximum value is 0.

8.



a. The graph opens downward.

b. The vertex is $(0, 0)$.

c. The axis of symmetry is $x = 0$.

d. The domain is $x \in R$, and the range is $y \leq 0$.

e. The maximum value is 0.

15. The graph of $y = 3x^2$ is narrower than the graph of $y = x^2$.

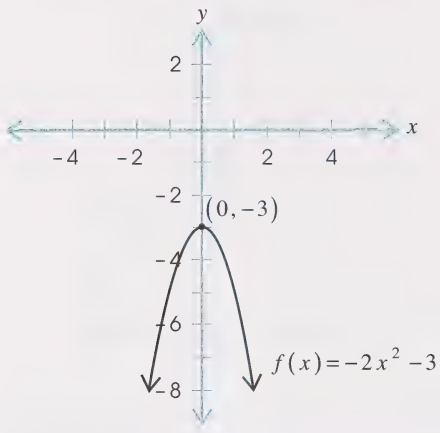
16. The graph of $y = -\frac{1}{3}x^2$ is wider than the graph of $y = -x^2$.

18. The graph of $y = -0.25x^2$ is the reflection of the graph of $y = 0.25x^2$ in the x-axis.

Section 1: Activity 1 (continued)

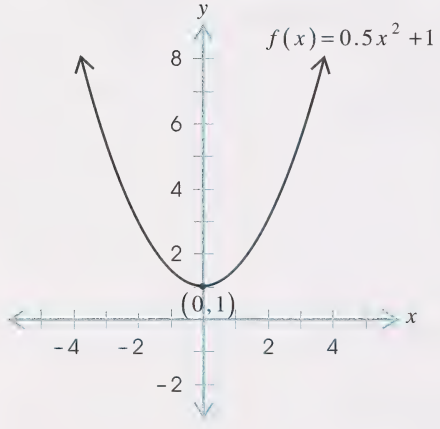
4. Textbook questions 9, 10, 17, 19, 21, 25, 48, 49, 54, 57, and 61 of "Practice," pp. 109 and 110

9.



a. The graph opens downward.
b. The vertex is $(0,-3)$.
c. The axis of symmetry is $x=0$.
d. The domain is $x \in R$, and the range is $y \leq -3$.
e. The maximum value is -3.

10.



a. The graph opens upward.
b. The vertex is $(0,1)$.
c. The axis of symmetry is $x=0$.
d. The domain is $x \in R$, and the range is $y \geq 1$.
e. The minimum value is 1.

17. The graph of $y=2x^2+7$ is the graph of $y=2x^2-2$ translated 9 units upward.

19. **a.** $y=-2x^2+3$ **b.** $y=2x^2-3$ **c.** $y=2x^2+3$ **d.** $y=-2x^2-3$

21. **a.** The graph opens upward.
b. The vertex is $(0,-11.4)$.
c. The axis of symmetry is $x=0$.
d. The domain is $x \in R$, and the range is $y \geq -11.4$.
e. The minimum value is -11.4.

25. $y-9.9=1.6x^2$

$y=1.6x^2+9.9$

a. The graph opens upward.

b. The vertex is $(0,9.9)$.

c. The axis of symmetry is $x=0$.

d. The domain is $x \in R$, and the range is $y \geq 9.9$.

e. The minimum value is 9.9.

48. Because the vertex is at $(0,0)$, $q=0$. Therefore, substitute $a=5$ and $q=0$ into $y=ax^2+q$.

$y=ax^2+q$

$=5x^2+0$

$=5x^2$

49. Because the vertex is at $(0,0)$, $q=0$. Therefore, substitute $a=-6$ and $q=0$ into $y=ax^2+q$.

$y=ax^2+q$

$=-6x^2+0$

$=-6x^2$

54. Because the vertex is at $(0,0)$, $q=0$.

$\therefore\ y=ax^2+q$

$=ax^2+0$

$=ax^2$

Because $(2,16)$ lies on the graph, substitute $x=2$ and $y=16$ into $y=ax^2$ to find a.

$16=a(2)^2$

$16=4a$

$a=4$

$\therefore\ y=4x^2$

Section 1: Activity 1 (continued)

57. Because the vertex is at $(0, 3)$, $q = 3$.

$$\therefore y = ax^2 + q$$
$$= ax^2 + 3$$

Because $(4, -13)$ lies on the graph, substitute $x = 4$ and $y = -13$ into $y = ax^2 + 3$ to find a.

$$-13 = a(4)^2 + 3$$
$$-13 = 16a + 3$$
$$-16 = 16a$$
$$a = -1$$

$$\therefore y = -1x^2 + 3$$
$$= -x^2 + 3$$

61. Because the vertex lies at $(0, -2)$, $q = -2$.

$$\therefore y = ax^2 + q$$
$$= ax^2 - 2$$

Because $(-4, -10)$ lies on the graph, substitute $x = -4$ and $y = -10$ into $y = ax^2 - 2$ to find a.

$$-10 = a(-4)^2 - 2$$
$$-10 = 16a - 2$$
$$-8 = 16a$$
$$a = -\frac{1}{2}$$

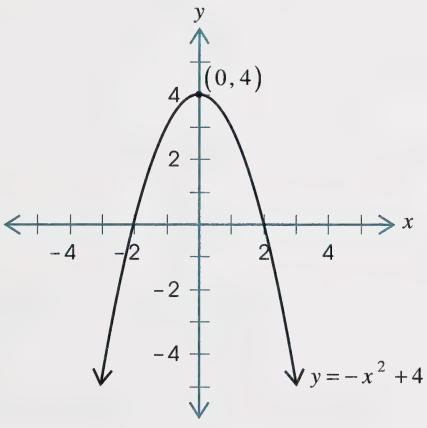
$$\therefore y = -\frac{1}{2}x^2 - 2$$

5. a. Textbook questions 28, 30, 34, 36, and 43 of "Practice," p. 109

28. The vertical stretch of scale factor 2 maps $(2,4)$ onto $(2,4\times2)$ or $(2,8)$. The vertical translation of 5 maps $(2,8)$ onto $(2,8+5)$ or $(2,13)$.

30. A reflection in the x-axis maps $(2,4)$ onto $(2,-4)$. A vertical stretch of scale factor $\frac{1}{2}$ maps $(2,-4)$ onto $\left(2,\frac{1}{2}(-4)\right)$ or $(2,-2)$.

34.



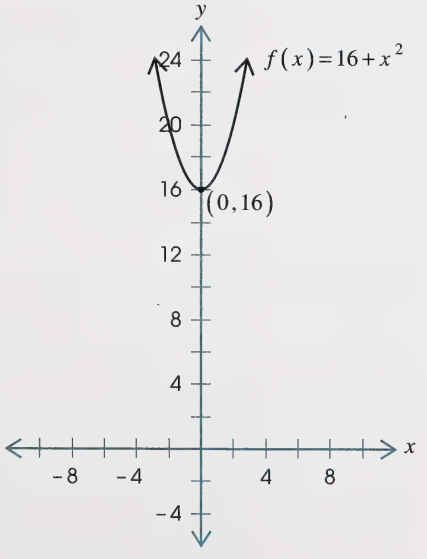
a. The vertex is $(0,4)$.

b. To determine the x-intercepts, let $y=0$.

$$\therefore\ y = -x^2 + 4$$
$$0 = -x^2 + 4$$
$$x^2 = 4$$
$$x = \pm\sqrt{4}$$
$$= \pm 2$$

The x-intercepts are ± 2.

36.

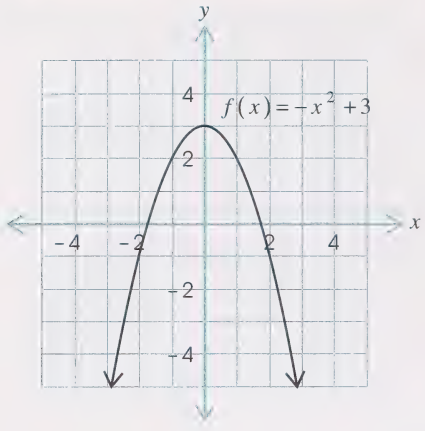


a. The vertex is $(0,16)$.

b. There are no x-intercepts.

Section 1: Activity 1 (continued)

43.



The x-intercepts are approximately ± 1.7.

b. Textbook questions 64, 65, 67 to 71, 72.a., and 76 of "Applications and Problem Solving," pp. 110 and 111

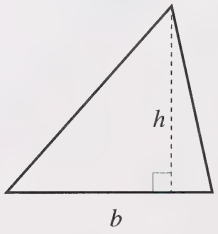
64. Because $(-3, -33)$ lies on the graph, substitute $x = -3$ and $y = -33$ into the equation.

$$\begin{aligned} y &= -2x^2 + c \\ -33 &= -2(-3)^2 + c \\ -33 &= -2(9) + c \\ -33 &= -18 + c \\ -15 &= c \\ c &= -15 \end{aligned}$$

65. The graph of $f(x) = ax^2 + q$ is symmetrical with respect to the y-axis. Therefore, if one x-intercept is 7.5, the other is -7.5.

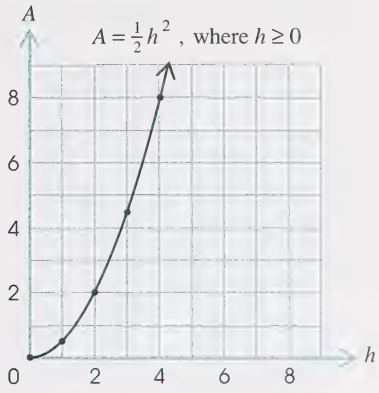
67. a. If $b = h$, then

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}h \bullet h \\ &= \frac{1}{2}h^2, \text{ where } h \geq 0 \end{aligned}$$



b.

h	A
0	0
1	0.5
2	2
3	4.5
4	8

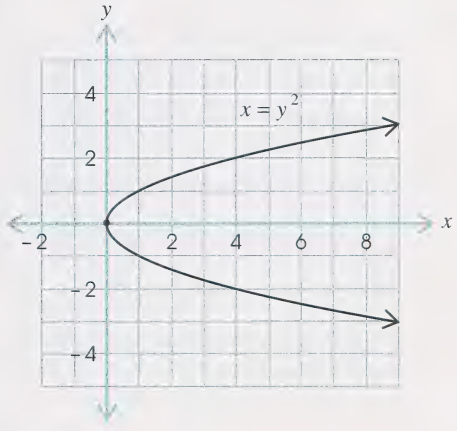


c. The h-intercept is 0.
The A-intercept is 0.

d. The domain is $h \geq 0$, and the range is $A \geq 0$.

68. a.

x	y
0	0
1	± 1
4	± 2
9	± 3

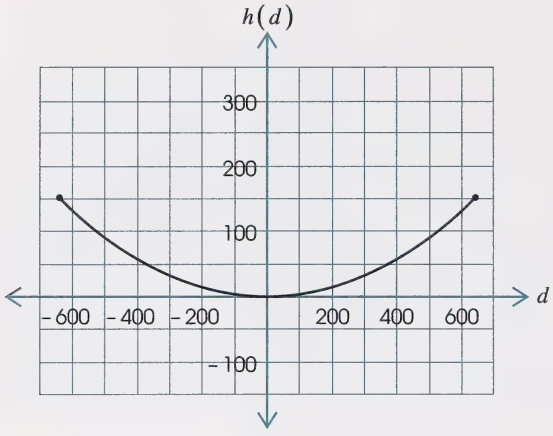


b. The domain is $x \geq 0$, and the range is $y \in R$.

c. The relation is not a function. The ordered pairs $(4, 2)$ and $(4, -2)$, for example, lie on the same vertical line. This relation fails the vertical line test for a function.

Section 1: Activity 1 (continued)

69. a.



b. The vertex $(0, 2)$ of the graph is its minimum point. Therefore, the cable is 2 m above the road at its lowest point.

c. The height of the towers can be determined by substituting $d = \pm\frac{1280}{2}$ or ± 640 into the function.

$$h(d) = 0.000\,37d^2 + 2$$
$$h(\pm 640) = 0.000\,37(\pm 640)^2 + 2$$
$$= 153.552$$

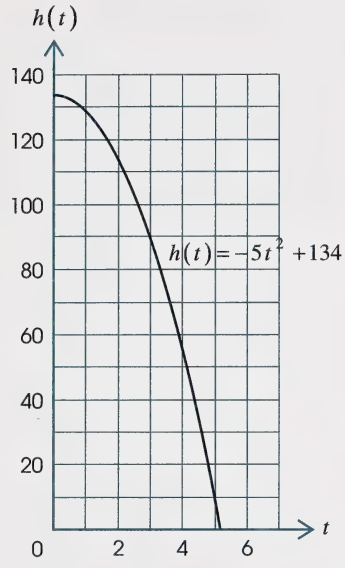
The maximum height of the towers, correct to the nearest ten metres, is 150 m.

d. $h(d) = 0.000\,37d^2 + 2$

$$h(200) = 0.000\,37(200)^2 + 2$$
$$= 16.8$$

At a horizontal distance of 200 m from the centre of the bridge, the cable is approximately 17 m above the road.

70. a.



b. The graph was drawn in the first quadrant because both time, t, and height, $h(t)$, are non-negative.

c. The object appears to strike the ground in about 5.2 s.

To determine the time algebraically, let $h(t) = 0$.

$$\therefore\ 0 = -5t^2 + 134$$
$$5t^2 = 134$$
$$t^2 = 26.8$$
$$t = \sqrt{26.8}$$
$$\doteq 5.2 \text{ s}$$

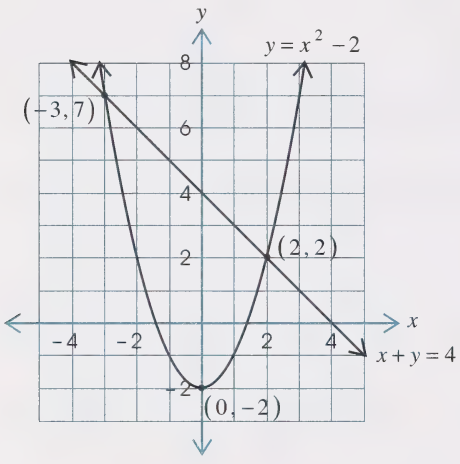
Section 1: Activity 1 (continued)

d. Let $h(t) = 0$ and $d = 134$.

$$\begin{aligned} \therefore\ h(t) &= -0.8t^2 + d \\ 0 &= -0.8t^2 + 134 \\ 0.8t^2 &= 134 \\ t^2 &= \frac{134}{0.8} \\ t^2 &= 167.5 \\ t &= \sqrt{167.5} \\ &\doteq 12.9 \end{aligned}$$

The object, if dropped from a height of 134 m on the Moon, would take approximately 12.9 s to reach the Moon's surface.

71.



a. From the graph, the curves appear to intersect at $(2, 2)$ and $(-3, 7)$.

b. The points of intersection can be found algebraically by solving the system of equations.

$x+y=4$ (1)

$y=x^2-2$ (2)

Substitute x^2-2 for y into (1).

$$x+\left(x^2-2\right)=4$$
$$x^2+x-2=4$$
$$x^2+x-6=0$$
$$(x+3)(x-2)=0$$

$\therefore\ x+3=0$ or $x-2=0$

$x=-3$ $\quad$ $x=2$

Substitute $x=-3$ and $x=2$ into (2).

For $x=-3$,

$$y=(-3)^2-2$$
$$=9-2$$
$$=7$$

For $x=2$,

$$y=(2)^2-2$$
$$=4-2$$
$$=2$$

Therefore, the graphs intersect each other at $(-3,7)$ and at $(2,2)$.

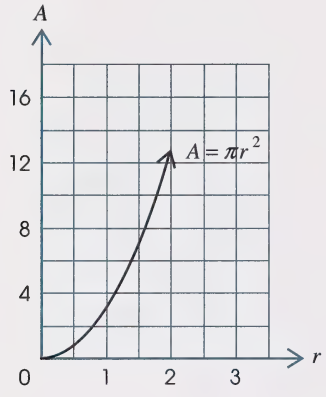
72. a. The area, A, of the building is, at most, s^2.

$\therefore\ A\le s^2$

Section 1: Activity 1 (continued)

76. a. $A = \pi r^2$

b.

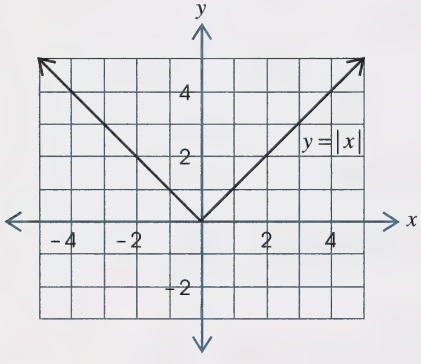


c. There is no axis of symmetry because the graph appears only in the first quadrant.

d. The domain is $r \geq 0$, and the range is $A \geq 0$.

6. a. Textbook questions 1 to 6 of Investigation 1, "Exploring $y = |x|$," p. 102

1.



2. The graph opens upward.

3. Because $y \geq 0$, the graph lies in the first and second quadrants only.

x	0	± 1	± 2	± 3	± 4
y	0	1	2	3	4

Absolute value is non-negative. For example, if $x = \pm 4$, then $y = |\pm 4| = 4$.

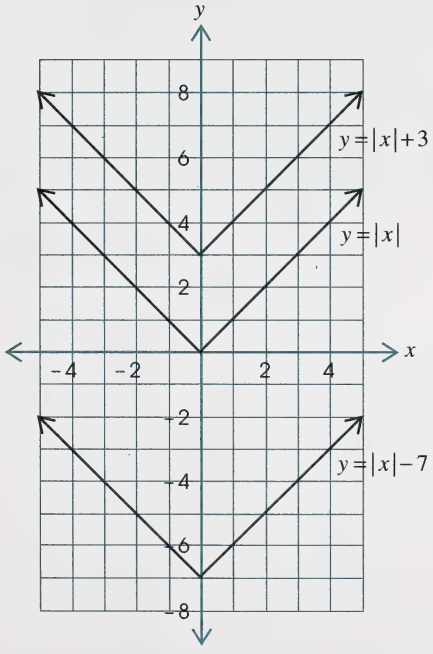
4. **a.** The axis of symmetry is the y-axis.
b. The equation of the y-axis of symmetry is $x = 0$.

5. The vertex is $(0, 0)$.

6. **a.** The domain is $x \in R$.
b. The range is $y \geq 0$.

b. Textbook questions 1 to 5 of Investigation 2, "Comparing $y = |x|$ and $y = |x| + q$," p. 102

1.



2. The three graphs are congruent, but the vertex of each graph is located at a different point.

Section 1: Activity 1 (continued)

3.

		$y=\|x\|$	$y=\|x\|+3$	$y=\|x\|-7$
a.	Vertex	$(0,0)$	$(0,3)$	$(0,-7)$
b.	Axis of Symmetry	$x=0$	$x=0$	$x=0$
c.	Domain	$x \in R$	$x \in R$	$x \in R$
	Range	$y \geq 0$	$y \geq 3$	$y \geq -7$

4. a. A vertical translation of 3 units upward maps the graph of $y=|x|$ onto the graph of $y=|x|+3$.

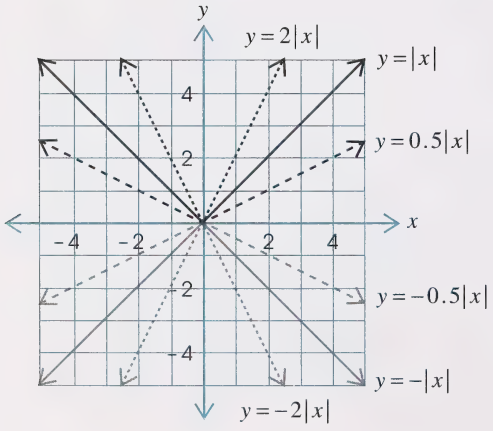
b. A vertical translation of 7 units downward maps the graph of $y=|x|$ onto the graph of $y=|x|-7$.

5.

	Graph	Vertex	Domain	Range
a.	$y=\|x\|+11$	$(0,11)$	$x \in R$	$y \geq 11$
b.	$y=\|x\|-10$	$(0,-10)$	$x \in R$	$y \geq -10$
c.	$y=\|x\|-22$	$(0,-22)$	$x \in R$	$y \geq -22$
d.	$y=\|x\|+44$	$(0,44)$	$x \in R$	$y \geq 44$

c. Textbook questions 1 to 6 of Investigation 5, "Comparing $y=|x|$ and $y=a|x|$," p. 103

1.



2. a. The graphs of $y = 0.5|x|$, $y = |x|$, and $y = 2|x|$ open upward; the graphs of $y = -0.5|x|$, $y = -|x|$, and $y = -2|x|$ open downward.

b. The sign of a determines the direction of opening. If $a > 0$, the graph of $y = a|x|$ opens upward. If $a < 0$, the graph of $y = a|x|$ opens downward.

3.

		$y = \|x\|$	$y = -\|x\|$	$y = 2\|x\|$	$y = -2\|x\|$	$y = 0.5\|x\|$	$y = -0.5\|x\|$
a.	Vertex	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$
b.	Axis of Symmetry	$x = 0$	$x = 0$	$x = 0$	$x = 0$	$x = 0$	$x = 0$
c.	Domain	$x \in R$	$x \in R$	$x \in R$	$x \in R$	$x \in R$	$x \in R$
	Range	$y \geq 0$	$y \leq 0$	$y \geq 0$	$y \leq 0$	$y \geq 0$	$y \leq 0$

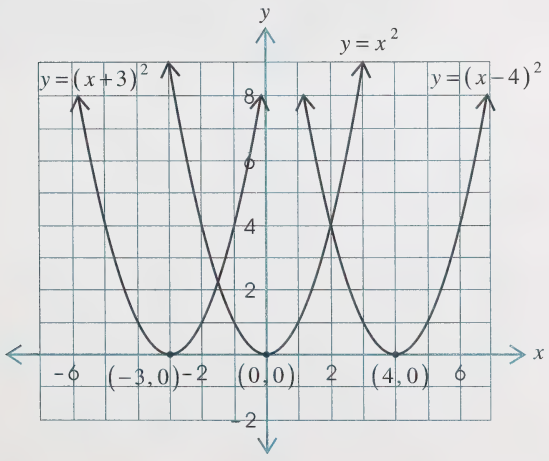
4. A reflection across the x-axis maps $y = |x|$ onto $y = -|x|$, maps $y = 2|x|$ onto $y = -2|x|$, and maps $y = 0.5|x|$ onto $y = -0.5|x|$.

5. The graphs of $y = 2|x|$ and $y = -2|x|$ are narrower than the graphs of $y = |x|$ and $y = -|x|$.

6. The graphs of $y = 0.5|x|$ and $y = -0.5|x|$ are wider than the graphs of $y = |x|$ and $y = -|x|$.

Section 1: Activity 2

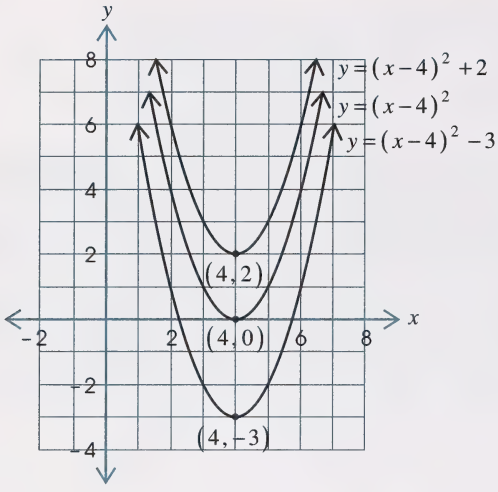
1. a. Textbook question "Explore: Compare the Graphs," p. 114



Group 1

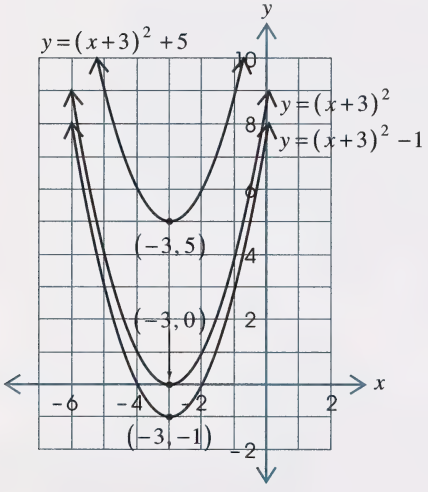
	Function	Vertex	Axis of Symmetry
a.	$y = x^2$	$(0,0)$	$x = 0$
b.	$y = (x-4)^2$	$(4,0)$	$x = 4$
c.	$y = (x+3)^2$	$(-3,0)$	$x = -3$

Section 1: Activity 2 (continued)



Group 2

	Function	Vertex	Axis of Symmetry
a.	$y=(x-4)^2$	$(4,0)$	$x=4$
b.	$y=(x-4)^2+2$	$(4,2)$	$x=4$
c.	$y=(x-4)^2-3$	$(4,-3)$	$x=4$



Group 3

	Function	Vertex	Axis of Symmetry
a.	$y=(x+3)^2$	$(-3,0)$	$x=-3$
b.	$y=(x+3)^2+5$	$(-3,5)$	$x=-3$
c.	$y=(x+3)^2-1$	$(-3,-1)$	$x=-3$

b. Textbook questions 1 to 6 of "Inquire," pp. 114 and 115

1. a. The graph of $y=x^2$ is translated 4 units to the right when x is replaced by $x-4$.

b. The graph of $y=x^2$ is translated 3 units to the left when x is replaced by $x+3$.

2. a. For $y=(x-7)^2$, the vertex is $(7,0)$ and the axis of symmetry is $x=7$.

b. For $y=(x+9)^2$, the vertex is $(-9,0)$ and the axis of symmetry is $x=-9$.

3. a. A vertical translation of 2 units upward maps the graph of $y=(x-4)^2$ onto the graph of $y=(x-4)^2+2$.

b. A vertical translation of 3 units downward maps the graph of $y=(x-4)^2$ onto the graph of $y=(x-4)^2-3$.

4. a. For $y=(x-7)^2+6$, the vertex is $(7,6)$ and the axis of symmetry is $x=7$.

b. For $y=(x-6)^2-5$, the vertex is $(6,-5)$ and the axis of symmetry is $x=6$.

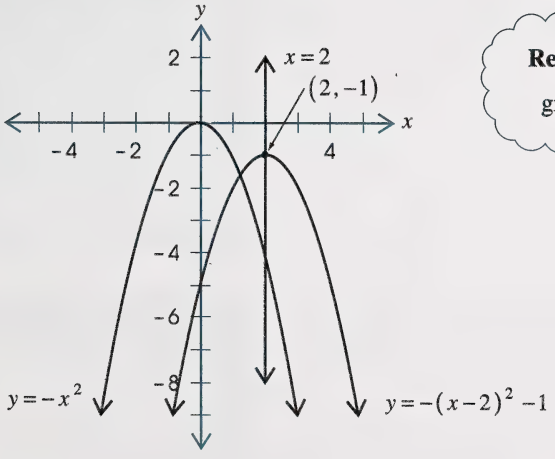
5. a. A vertical translation of 5 units upward maps the graph of $y=(x+3)^2$ onto the graph of $y=(x+3)^2+5$.

b. A vertical translation of 1 unit downward maps the graph of $y=(x+3)^2$ onto the graph of $y=(x+3)^2-1$.

6. a. For $y=(x+9)^2+4$, the vertex is $(-9,4)$ and the axis of symmetry is $x=-9$.

b. For $y=(x+5)^2-7$, the vertex is $(-5,-7)$ and the axis of symmetry is $x=-5$.

2. a. The graph of $y=-(x-2)^2-1$ is the graph of $y=-x^2$ translated 2 units to the right and 1 unit downward.



Remember: The graph of $y=-x^2$ is the graph of $y=x^2$ reflected in the x-axis.

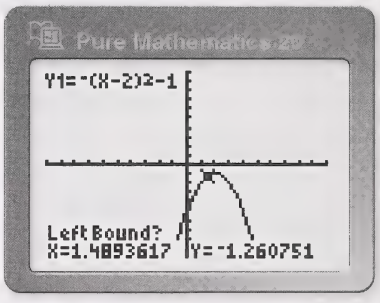
Section 1: Activity 2 (continued)

b. Before starting, be sure you are using the standard window settings. First, graph the function by pressing the following:

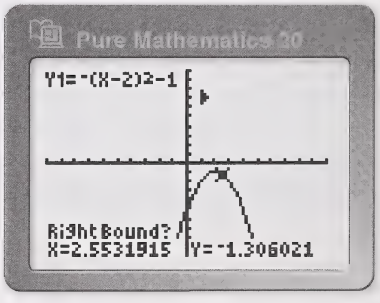
Y= (–) (X,T,Θ,*n* – 2) x^2 – 1 GRAPH

Because the graph of $y = -(x-2)^2 - 1$ opens downward, the function has a maximum value. Therefore, you must use the Maximum feature in the CALCULATE menu. To determine the maximum, press 2nd [CALC] 4.

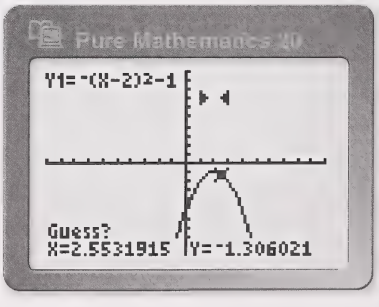
Use the arrow keys to move the cursor along the graph until it is close to, but to the left of the vertex (as shown in the display); then press ENTER.



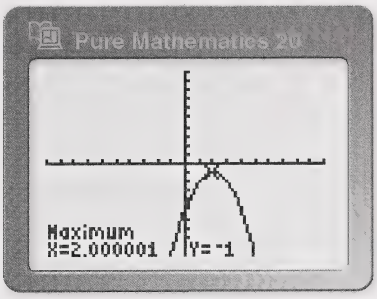
Use the arrow keys to position the cursor to the right of the vertex (as shown in the display); then press ENTER.



To enter your guess, press the left arrow key once to position the cursor between the left bound and right bound and press ENTER.



Your display should look as follows:

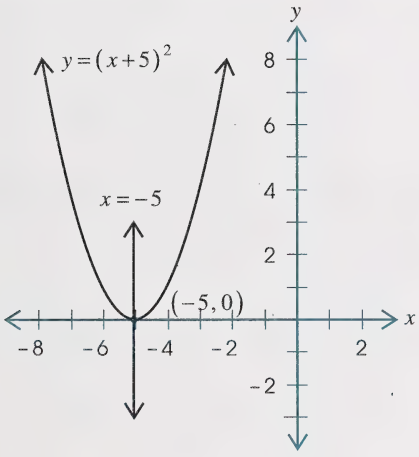


Note: Small rounding errors may occur.

The vertex is $(2,-1)$, and the axis of symmetry is $x=2$.

3. Textbook questions 1, 2, 4, 5, 13, 14, 23, and 24 of "Practice," p. 118

1.



a. The graph opens upward.

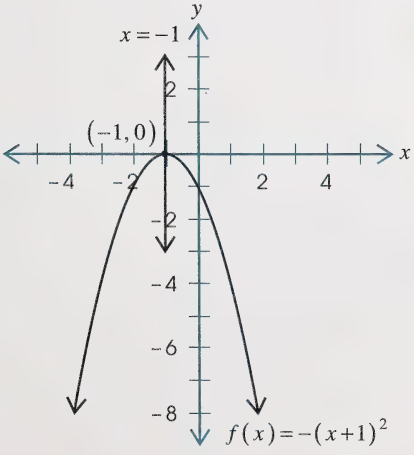
b. The vertex is $(-5,0)$.

c. The axis of symmetry is $x=-5$.

d. The domain is $x \in R$, and the range is $y \geq 0$.

e. The minimum is 0.

2.



a. The parabola opens downward.

b. The vertex is $(-1,0)$.

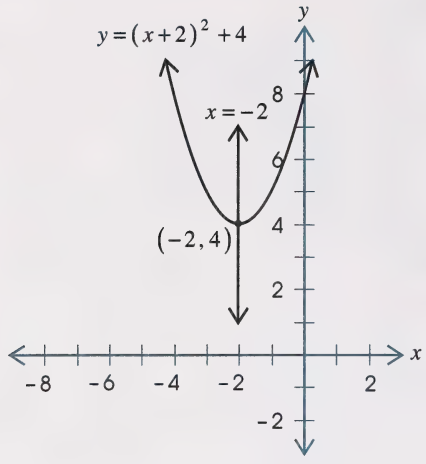
c. The axis of symmetry is $x=-1$.

d. The domain is $x \in R$, and the range is $y \leq 0$.

e. The maximum is 0.

Section 1: Activity 2 (continued)

4.



a. The parabola opens upward.

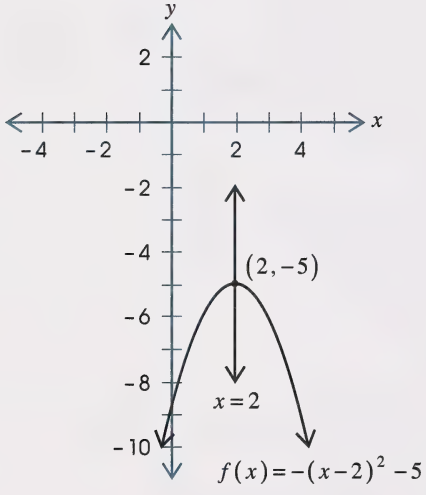
b. The vertex is $(-2, 4)$.

c. The axis of symmetry is $x=-2$.

d. The domain is $x \in R$, and the range is $y \geq 4$.

e. The minimum is 4.

5.



a. The parabola opens downward.

b. The vertex is $(2, -5)$.

c. The axis of symmetry is $x=2$.

d. The domain is $x \in R$, and the range is $y \leq -5$.

e. The maximum is $y=-5$.

13. **a.** The parabola opens upward.

b. The vertex is $(2, 1)$.

c. The axis of symmetry is $x=2$.

d. The domain is $x \in R$, and the range is $y \geq 1$.

e. The minimum is 1.

14. **a.** The parabola opens downward.

b. The vertex is $(-1, -2)$.

c. The axis of symmetry is $x = -1$.

d. The domain is $x \in R$, and the range is $y \leq -2$.

e. The maximum is -2.

23. **a.** The parabola opens upward.

b. The parabola is narrower than $y = x^2$.

c. The vertex is $(-1.5, -9)$.

d. The axis of symmetry is $x = -1.5$.

e. The minimum is -9.

24. **a.** The parabola opens downward.

b. The parabola is narrower than $y = x^2$.

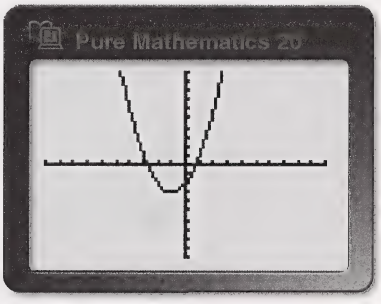
c. The vertex is $(2.6, 3.3)$.

d. The axis of symmetry is $x = 2.6$.

e. The maximum is 3.3.

4. a. Textbook questions 38, 40, 48, 52, 55, 59, and 64 of "Practice," p. 119

38. The x-intercepts are approximately -2.7 and 0.7.
The y-intercept is -2.



These values may be confirmed algebraically.

To find the x-intercepts, let $y = 0$.

$$\therefore y = (x+1)^2 - 3$$
$$0 = (x+1)^2 - 3$$
$$(x+1)^2 = 3$$
$$x + 1 = \pm\sqrt{3}$$
$$x = -1 \pm \sqrt{3}$$

$$\therefore x = -1 + \sqrt{3} \quad \text{or} \quad x = -1 - \sqrt{3}$$
$$\doteq 0.7 \qquad\qquad \doteq -2.7$$

The x-intercepts are approximately -2.7 and 0.7.

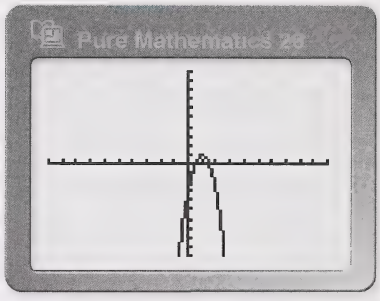
Section 1: Activity 2 (continued)

To find the y-intercept, let $x = 0$.

$$\begin{aligned}\therefore\ y &= (x+1)^2 - 3\\ &= (0+1)^2 - 3\\ &= 1-3\\ &= -2\end{aligned}$$

The y-intercept is -2.

40. To find the x-intercepts, let $y = 0$.



$$\begin{aligned}\therefore\ y &= -4(x-1)^2 + 1\\ 0 &= -4(x-1)^2 + 1\\ 4(x-1)^2 &= 1\\ (x-1)^2 &= \frac{1}{4}\\ x-1 &= \pm\sqrt{\frac{1}{4}}\\ x-1 &= \pm\frac{1}{2}\\ x &= 1\pm\frac{1}{2}\end{aligned}$$

$$\therefore\ x = 1+\frac{1}{2} \quad \text{or} \quad x = 1-\frac{1}{2}$$

$$= 1\frac{1}{2} \qquad\qquad = \frac{1}{2}$$

The x-intercepts are $\frac{1}{2}$ and $1\frac{1}{2}$.

To find the y-intercept, let $x = 0$.

$$\begin{aligned}\therefore\ y &= -4(x-1)^2 + 1\\ &= -4(0-1)^2 + 1\\ &= -4(1)+1\\ &= -3\end{aligned}$$

The y-intercept is -3.

48. Because the vertex is $(3,-5)$, $p=3$ and $q=-5$. Substitute $a=2$, $p=3$, and $q=-5$ into the standard form.

$$\begin{aligned} y &= a(x-p)^2+q \\ &= 2(x-3)^2+(-5) \\ &= 2(x-3)^2-5 \end{aligned}$$

52. Because the required curve is congruent to $y=x^2$ and opens upward, $a=1$.

Because the vertex is $(1,5)$, $p=1$ and $q=5$. Substitute $a=1$, $p=1$, and $q=5$ into the standard form.

$$\begin{aligned} y &= a(x-p)^2+q \\ &= 1(x-1)^2+5 \\ &= (x-1)^2+5 \end{aligned}$$

55. Because the required curve is congruent to $y=2x^2$ and has a maximum value, $a=-2$.

Because the maximum point is $(2,-3)$, $p=2$ and $q=-3$. Substitute $a=-2$, $p=2$, and $q=-3$ into the standard form.

$$\begin{aligned} y &= a(x-p)^2+q \\ &= -2(x-2)^2-3 \end{aligned}$$

59. Because the required curve is congruent to $y=2x^2$ and has a minimum value, $a=2$.

Because the minimum value is -6 and the axis of symmetry is $x=-5$, the vertex is $(-5,-6)$. Therefore, $p=-5$ and $q=-6$.

Substitute $a=2$, $p=-5$, and $q=-6$ into the standard form.

$$\begin{aligned} y &= a(x-p)^2+q \\ &= 2\left[x-(-5)\right]^2-6 \\ &= 2(x+5)^2-6 \end{aligned}$$

Section 1: Activity 2 (continued)

64. Because the vertex is $(-5,-3)$, $p=-5$ and $q=-3$. Substitute $p=-5$ and $q=-3$ into the standard form.

$$\begin{aligned} y &= a(x-p)^2+q \\ &= a\left[x-(-5)\right]^2+(-3) \\ &= a(x+5)^2-3 \end{aligned}$$

Now, substitute the coordinates of the given point, $(-3,-11)$, into the equation to find a.

$$\begin{aligned} y &= a(x+5)^2-3 \\ -11 &= a(-3+5)^2-3 \\ -11 &= a(2)^2-3 \\ -8 &= 4a \\ 4a &= -8 \\ a &= -2 \end{aligned}$$

$\therefore\ y=-2(x+5)^2-3$

b. Textbook questions 71 to 75, 79, 81, 86, and 88 of "Applications and Problem Solving," pp. 119 to 121

71. Because the vertex is $(-2,-4)$, $p=-2$ and $q=-4$. Substitute $p=-2$ and $q=-4$ into the standard form.

$$\begin{aligned} y &= a(x-p)^2+q \\ &= a\left[x-(-2)\right]^2+(-4) \\ &= a(x+2)^2-4 \end{aligned}$$

If one x-intercept is 7, the graph passes through $(7,0)$. Substitute $(7,0)$ into the equation to find a.

$$y = a(x+2)^2 - 4$$
$$0 = a(7+2)^2 - 4$$
$$0 = a(9)^2 - 4$$
$$4 = 81a$$
$$a = \frac{4}{81}$$

$$\therefore\ y = \frac{4}{81}(x+2)^2 - 4$$

To determine the x-intercepts, let $y = 0$.

$$\therefore\ y = \frac{4}{81}(x+2)^2 - 4$$
$$0 = \frac{4}{81}(x+2)^2 - 4$$
$$4 = \frac{4}{81}(x+2)^2$$
$$4(81) = 4(x+2)^2$$
$$\frac{4(81)}{4} = (x+2)^2$$
$$(x+2)^2 = 81$$
$$x+2 = \pm\sqrt{81}$$
$$x+2 = \pm 9$$

$$\therefore\ x+2 = 9 \quad \text{or} \quad x+2 = -9$$
$$x = 7 \qquad\qquad x = -11$$

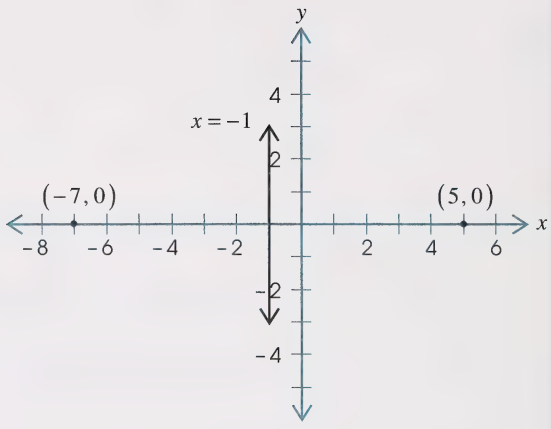
Therefore, the other x-intercept is -11.

Section 1: Activity 2 (continued)

72. The axis of symmetry is midway between $x = -7$ and $x = 5$.

$$\therefore x = \frac{-7+5}{2}$$
$$= \frac{-2}{2}$$
$$= -1$$

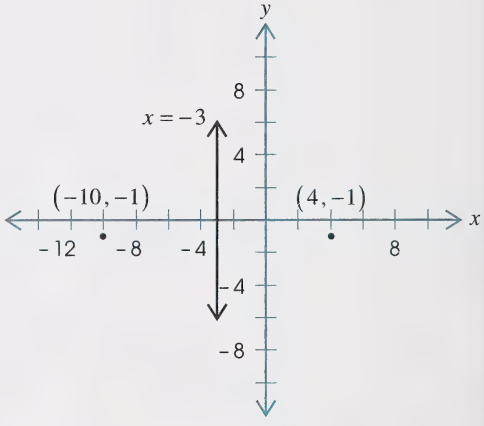
The axis of symmetry is $x = -1$.



73. The axis of symmetry is midway between $(-10, -1)$ and $(4, -1)$.

$$\therefore x = \frac{-10+4}{2}$$
$$= \frac{-6}{2}$$
$$= -3$$

The axis of symmetry is $x = -3$.



74. **a.** The vertex of $h(t) = -9(t-3)^2 + 83$ is $(3, 83)$. Therefore, the maximum height of the flare is 83 m.

b. The maximum height occurs at $t = 3$. This means that the flare rises for 3 s and falls for approximately 3 s. Therefore, the flare burns for about 6 s before it hits the water. The total time is an approximation, because the flare was fired from a height of 2 m.

75. Substitute $(2,6)$ into the standard form.

$$\begin{aligned} y &= a(x-p)^2 + q \\ 6 &= a(2-1)^2 + q \\ 6 &= a(1) + q \\ a + q &= 6 \quad (1) \end{aligned}$$

Substitute $(3,12)$ into the standard form.

$$\begin{aligned} y &= a(x-p)^2 + q \\ 12 &= a(3-1)^2 + q \\ 12 &= a(2)^2 + q \\ 4a + q &= 12 \quad (2) \end{aligned}$$

Use (1) and (2) to solve for a and q.

$$\begin{array}{rl} a + q = 6 & (1) \\ 4a + q = 12 & (2) \\ \hline -3a = -6 & (1) - (2) \\ a = 2 & \end{array}$$

Substitute $a = 2$ into (1).

$$\begin{aligned} a + q &= 6 \\ 2 + q &= 6 \\ q &= 4 \end{aligned}$$

79. a. The vertex is $(2.5, 38.5)$; therefore, the maximum height of the ball is 38.5 m.

b. The ball was hit at $t = 0$ s.

$$\begin{aligned} \therefore h(t) &= -6(t-2.5)^2 + 38.5 \\ h(0) &= -6(0-2.5)^2 + 38.5 \\ &= -6(-2.5)^2 + 38.5 \\ &= -37.5 + 38.5 \\ &= 1 \end{aligned}$$

The ball was hit at a height of 1 m.

Section 1: Activity 2 (continued)

c. Method 1: Using Symmetry

The ball reaches its maximum height at $t = 2.5$ s. It will fall back down to the ground in slightly more than 2.5 s, since it must fall through an additional 1 m of height. Therefore, the ball is in the air for slightly more than 5 s.

Method 2: Solving an Equation

When the ball strikes the ground, $h(t) = 0$.

$$\therefore h(t) = -6(t-2.5)^2 + 38.5$$
$$0 = -6(t-2.5)^2 + 38.5$$
$$6(t-2.5)^2 = 38.5$$
$$(t-2.5)^2 = \frac{38.5}{6}$$
$$t - 2.5 = \pm\sqrt{\frac{38.5}{6}}$$

$$\therefore t = 2.5 + \sqrt{\frac{38.5}{6}} \quad \text{or} \quad t = 2.5 - \sqrt{\frac{38.5}{6}}$$
$$\doteq 5.033\,114\,026\ldots \qquad \doteq -0.033\,114\,025\,6\ldots$$

Because time is non-negative, the ball is in the air for appromimately 5 s.

d. $h(t) = -6(t-2.5)^2 + 38.5$

$$h(1) = -6(1-2.5)^2 + 38.5$$
$$= -6(-1.5)^2 + 38.5$$
$$= 25$$

The ball is 25 m above the ground after 1 s.

81. a. The domain is $s \geq 0$, and the range is $d \geq 0$.

b. To find the d-intercept, let $s = 0$.

$$\begin{aligned}\therefore d &= 0.006(s+15)^2 - 1.35 \\ &= 0.006(0+15)^2 - 1.35 \\ &= 0.006(15)^2 - 1.35 \\ &= 0\end{aligned}$$

To find the s-intercept, let $d = 0$.

$$\begin{aligned}\therefore d &= 0.006(s+15)^2 - 1.35 \\ 0 &= 0.006(s+15)^2 - 1.35 \\ 0.006(s+15)^2 &= 1.35 \\ (s+15)^2 &= \frac{1.35}{0.006} \\ (s+15)^2 &= 225 \\ s+15 &= \pm\sqrt{225} \\ s+15 &= \pm 15\end{aligned}$$

$$\therefore \quad s+15 = 15 \quad \text{or} \quad s+15 = -15$$
$$s = 0 \qquad\qquad s = -30$$

Because $s \geq 0$, the s-intercept is 0.

c.
$$\begin{aligned}d &= 0.006(s+15)^2 - 1.35 \\ &= 0.006(50+15)^2 - 1.35 \\ &= 0.006(65)^2 - 1.35 \\ &= 24\end{aligned}$$

$$\begin{aligned}d &= 0.006(s+15)^2 - 1.35 \\ &= 0.006(100+15)^2 - 1.35 \\ &= 0.006(115)^2 - 1.35 \\ &= 78\end{aligned}$$

The stopping distance at 50 km/h is 24 m; and the stopping distance at 100 km/h is 78 m.

d. Answers may vary. A sample answer is given.

The speed limit in a school zone is 30 km/h.

Section 1: Activity 2 (continued)

e. Answers may vary. A sample answer is given.

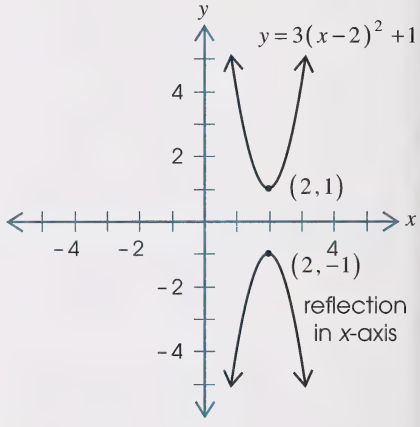
$$\begin{aligned} d &= 0.006(s+15)^2 - 1.35 \\ &= 0.006(30+15)^2 - 1.35 \\ &= 0.006(45)^2 - 1.35 \\ &= 10.8 \end{aligned}$$

The stopping distance on dry pavement in a school zone is 10.8 m.

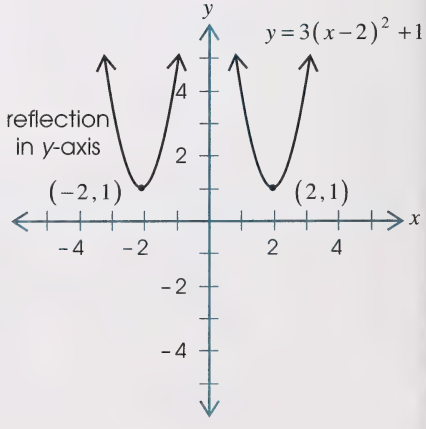
86. a. If the graphs both open up and are congruent, then $m = n$.
b. If the first graph is narrower than the second, then $m > n$.
c. If the first graph is wider than the second graph, then $m < n$.

88. a. The graph of $y = 3(x-2)^2 + 1$ opens upward from $(2,1)$.

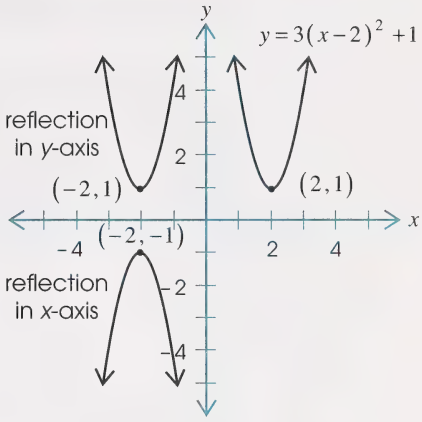
The mirror image in the x-axis will open downward from $(2,-1)$. Therefore, the equation of the image is $y = -3(x-2)^2 - 1$.



b. The reflection in the y-axis opens upward from $(-2,1)$. Therefore, the equation of the image is $y = 3(x+2)^2 + 1$.

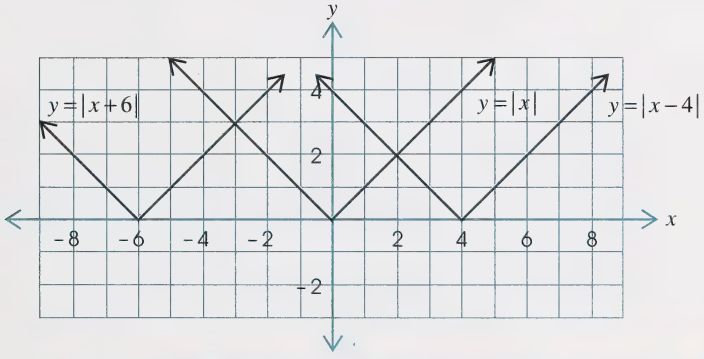


c. The curve that is obtained by a reflection in the y-axis and then a reflection in the x-axis opens downward from $(-2,-1)$. Therefore, the equation of the image is $y=-3(x+2)^2-1$.



5. a. Textbook questions 1 to 4 of Investigation 3, "Comparing $y=|x|$ and $y=|x-p|$," p. 102

1.



2.

		$y=\|x\|$	$y=\|x-4\|$	$y=\|x+6\|$
a.	Vertex	$(0,0)$	$(4,0)$	$(-6,0)$
b.	Axis of Symmetry	$x=0$	$x=4$	$x=-6$
c.	Domain	$x \in R$	$x \in R$	$x \in R$
	Range	$y \geq 0$	$y \geq 0$	$y \geq 0$

3. a. The graph of $y=|x|$ is translated 4 units to the right when x is replaced by $x-4$.

b. The graph of $y=|x|$ is translated 6 units to the left when x is replaced by $x+6$.

Section 1: Activity 2 (continued)

4. a. The vertex is $(9, 0)$, and the axis of symmetry is $x = 9$.

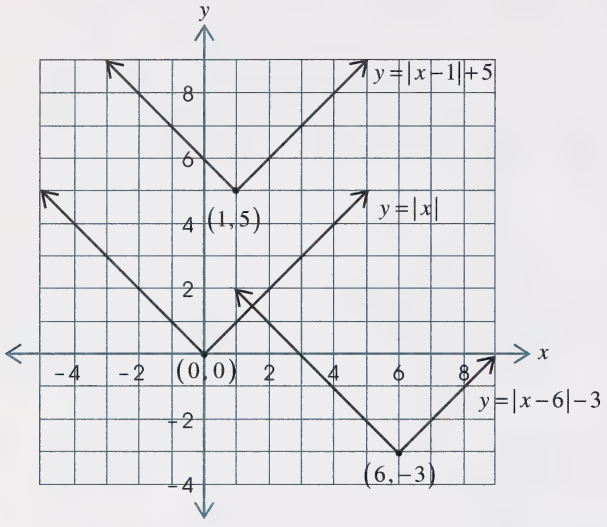
b. The vertex is $(-7, 0)$, and the axis of symmetry is $x = -7$.

c. The vertex is $(11, 0)$, and the axis of symmetry is $x = 11$.

d. The vertex is $(-13, 0)$, and the axis of symmetry is $x = -13$.

b. Textbook questions 1 to 3 of Investigation 4, "Comparing $y = |x|$ and $y = |x - p| + q$," p. 103

1.



2.

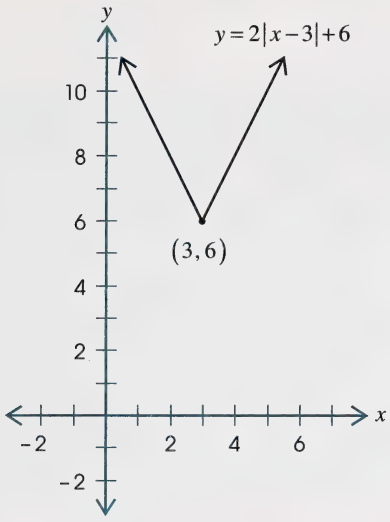
		$y = \|x\|$	$y = \|x - 1\| + 5$	$y = \|x - 6\| - 3$
a.	Vertex	$(0, 0)$	$(1, 5)$	$(6, -3)$
b.	Axis of Symmetry	$x = 0$	$x = 1$	$x = 6$
c.	Domain	$x \in R$	$x \in R$	$x \in R$
	Range	$y \geq 0$	$y \geq 5$	$y \geq -3$

3. a. A translation of 1 unit to the right and 5 units upward maps $y = |x|$ onto $y = |x - 1| + 5$.

b. A translation of 6 units to the right and 3 units downward maps $y = |x|$ onto $y = |x - 6| - 3$.

c. Textbook questions 4 and 8 of Investigation 6, "Combining Transformations," p. 103

4.



a. The vertex is $(3, 6)$.

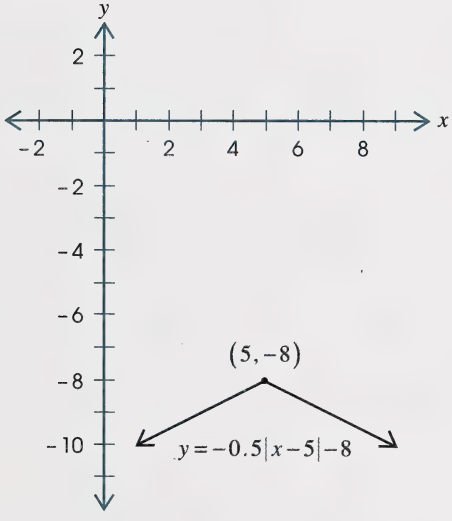
b. The axis of symmetry is $x = 3$.

c. The domain is $x \in R$, and the range is $y \geq 6$.

d. The graph opens upward.

e. The graph is narrower than $y = |x|$.

8.



a. The vertex is $(5, -8)$.

b. The axis of symmetry is $x = 5$.

c. The domain is $x \in R$, and the range is $y \leq -8$.

d. The graph opens downward.

e. The graph is wider than $y = -|x|$.

Section 1: Activity 3

1. a. Textbook question "Explore: Look for a Pattern," p. 126

Trinomial, x^2+bx+c	Value of b	Value of c	Factored Form, $(x-p)^2$	Value of p
x^2+6x+9	6	9	$(x+3)^2$	-3
x^2+2x+1	2	1	$(x+1)^2$	-1
$x^2+10x+25$	10	25	$(x+5)^2$	-5
x^2-2x+1	-2	1	$(x-1)^2$	1
$x^2-8x+16$	-8	16	$(x-4)^2$	4
$x^2-14x+49$	-14	49	$(x-7)^2$	7

b. Textbook questions 1 and 2 of "Inquire," p. 127

1. a. To find the value of c, square half the value of b.

b. To find p, factor the trinomial as the square of a binomial. The value of p is the constant term in the binomial changed in sign.

2. a. $x^2+12x+\mathbf{36}=(x+\mathbf{6})^2$; $p=\mathbf{-6}$

b. $x^2+16x+\mathbf{64}=(x+\mathbf{8})^2$; $p=\mathbf{-8}$

c. $x^2-20x+\mathbf{100}=(x-\mathbf{10})^2$; $p=\mathbf{10}$

d. $x^2-4x+\mathbf{4}=(x-\mathbf{2})^2$; $p=\mathbf{2}$

e. $x^2+1.6x+\mathbf{0.64}=(x-\mathbf{0.8})^2$; $p=\mathbf{0.8}$

f. $x^2-3x+\mathbf{\frac{9}{4}}=\left(x-\mathbf{\frac{3}{2}}\right)^2$; $p=\mathbf{\frac{3}{2}}$

2. Textbook questions 1, 5, 7, 9, 15, 16, 18, 19, 24, and 26 of "Practice," p. 131

1. Square half the coefficient of x.

$$c=\left(\frac{14}{2}\right)^2$$
$$=7^2$$
$$=49$$

$$\therefore\ x^2+14x+49=(x+7)^2$$

5. Square half the coefficient of x.

$$c = \left(-\frac{10}{2}\right)^2$$
$$= (-5)^2$$
$$= 25$$

$$\therefore\ x^2 - 10x + 25 = (x-5)^2$$

7. Square half the coefficient of x.

$$c = \left(-\frac{3}{2}\right)^2$$
$$= \frac{9}{4}$$

$$\therefore\ x^2 - 3x + \frac{9}{4} = \left(x - \frac{3}{2}\right)^2$$

9. Square half the coefficient of x.

$$c = \left(\frac{1}{2}\right)^2$$
$$= \frac{1}{4}$$

$$\therefore\ x^2 + x + \frac{1}{4} = \left(x + \frac{1}{2}\right)^2$$

15. Square half the coefficient of x.

$$c = \left[\frac{1}{2}\left(-\frac{2}{3}\right)\right]^2$$
$$= \left(-\frac{1}{3}\right)^2$$
$$= \frac{1}{9}$$

$$\therefore\ x^2 - \frac{2}{3}x + \frac{1}{9} = \left(x - \frac{1}{3}\right)^2$$

Section 1: Activity 3 (continued)

16. Square half the coefficient of x.

$$c = \left[\frac{1}{2}\left(\frac{1}{6}\right)\right]^2$$
$$= \left(\frac{1}{12}\right)^2$$
$$= \frac{1}{144}$$

$$\therefore\ x^2 + \frac{x}{6} + \frac{1}{144} = \left(x + \frac{1}{12}\right)^2$$

18. $y = x^2 - 4x - 1$
$= \left(x^2 - 4x + 4\right) - 4 - 1$ ← $\left(-\frac{4}{2}\right)^2 = 4$
$= \left(x^2 - 4x + 4\right) - 5$
$= (x-2)^2 - 5$

The equation in standard form is $y = (x-2)^2 - 5$.
Therefore, the vertex is $(2, -5)$, and the axis of symmetry is $x = 2$.

To find two other points on the graph, substitute suitable values of x into the equation.

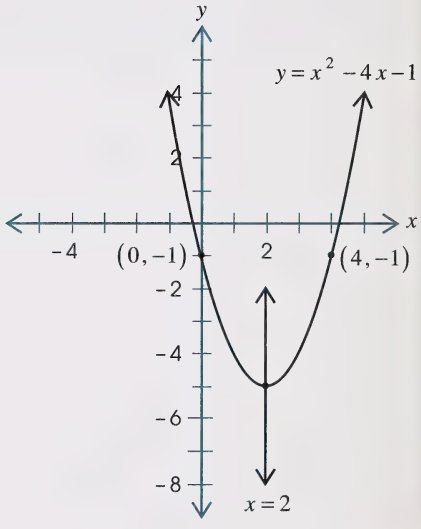
If $x = 0$, then

$y = x^2 - 4x - 1$
$= (0)^2 - 4(0) - 1$
$= -1$

If $x = 4$, then

$y = x^2 - 4x - 1$
$= (4)^2 - 4(4) - 1$
$= -1$

Two other points are $(0, -1)$ and $(4, -1)$.



19. $y = x^2 + 10x + 30$

$= (x^2 + 10x + 25) - 25 + 30$ ← $\left(\frac{10}{2}\right)^2 = 25$

$= (x+5)^2 + 5$

The vertex is $(-5, 5)$, and the axis of symmetry is $x = -5$.

If $x = -7$, then

$y = x^2 + 10x + 30$

$= (-7)^2 + 10(-7) + 30$

$= 9$

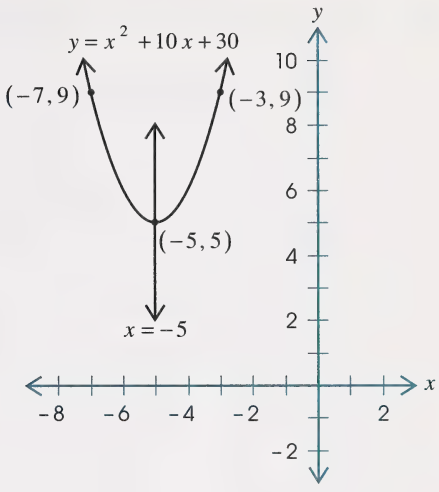
If $x = -3$, then

$y = x^2 + 10x + 30$

$= (-3)^2 + 10(-3) + 30$

$= 9$

Two other points are $(-7, 9)$ and $(-3, 9)$.



24. $y = x^2 - 2x - 8$

$= (x^2 - 2x + 1) - 1 - 8$

$= (x-1)^2 - 9$

The vertex is $(1, -9)$, and the axis of symmetry is $x = 1$.

To find the x-intercepts, let $y = 0$.

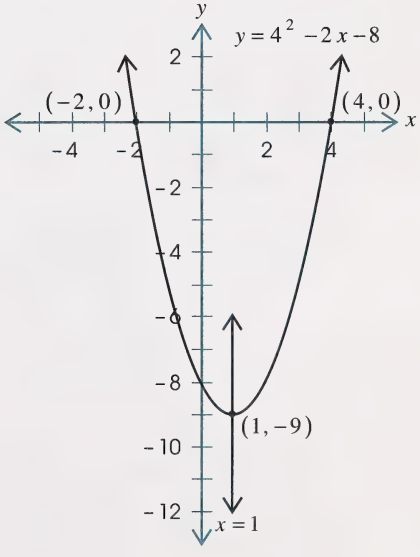
$\therefore\ y = x^2 - 2x - 8$

$0 = (x-4)(x+2)$

$\therefore\ x - 4 = 0$ or $x + 2 = 0$

$x = 4$ $x = -2$

The x-intercepts are -2 and 4.



Section 1: Activity 3 (continued)

To find the y-intercept, let $x=0$.

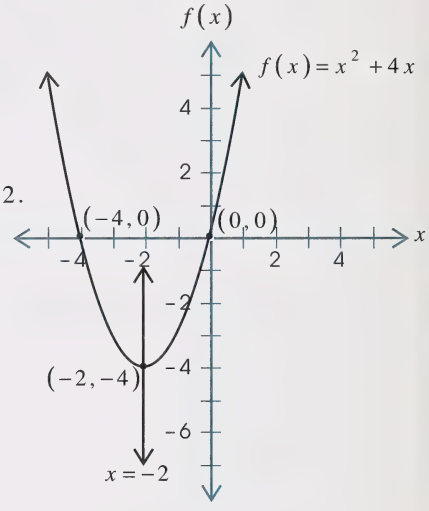
$$\therefore y = x^2 - 2x - 8$$
$$= (0)^2 - 2(0) - 8$$
$$= -8$$

The y-intercept is -8.

Because the graph opens upward and $(1,-9)$ is a minimum point, the range is $y \geq -9$.

26. $f(x) = x^2 + 4x$
$$= \left(x^2 + 4x + 4\right) - 4$$
$$= (x+2)^2 - 4$$

The vertex is $(-2,-4)$, and the axis of symmetry is $x=-2$.



To find the x-intercepts, let $f(x)=0$.

$$\therefore f(x) = x^2 + 4x$$
$$0 = x(x+4)$$

$$\therefore x = 0 \quad \text{or} \quad x+4=0$$
$$x = -4$$

The x-intercepts are -4 and 0.

To find the $f(x)$-intercept, let $x=0$.

$$\therefore f(x) = x^2 + 4x$$
$$f(0) = 0^2 + 4(0)$$
$$= 0$$

The $f(x)$-intercept is 0.

Because the graph opens upward and $(-2,-4)$ is a minimum point, the range is $f(x) \geq -4$.

3. Textbook questions 28, 33, 36, 38, 47, 49, 52, 54, 58, and 66 of "Practice," p. 131

28. $y = -x^2 + 8x - 11$

$= (-x^2 + 8x) - 11$ ← Group the terms containing x.

$= -1(x^2 - 8x) - 11$ ← Factor the coefficient of x^2 from the first two terms.

$= -1[(x^2 - 8x + 16) - 16] - 11$ ← Complete the square inside the parentheses.

$= -1[(x-4)^2 - 16] - 11$ ← Write the perfect-square trinomial as the square of a binomial.

$= -(x-4)^2 + 16 - 11$ ← Expand to remove the square brackets.

$= -(x-4)^2 + 5$ ← Simplify.

Therefore, the equation in standard form is
$y = -(x-4)^2 + 5$.

Using the equation in standard form, the vertex is (4, 5) and the axis of symmetry is $x = 4$.

Now, find two points on the graph.

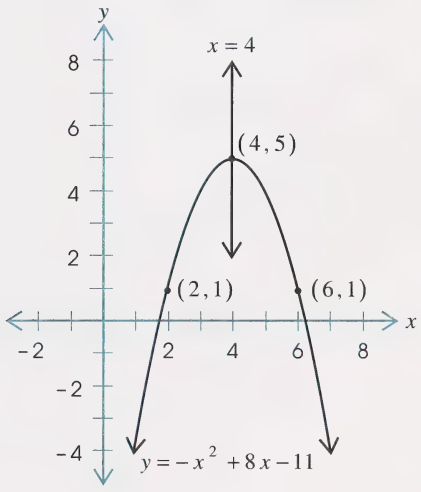
If $x = 2$, then

$y = -x^2 + 8x - 11$
$= -(2)^2 + 8(2) - 11$
$= 1$

If $x = 6$, then

$y = -x^2 + 8x - 11$
$= -(6)^2 + 8(6) - 11$
$= 1$

Two additional points are (2, 1) and (6, 1).



Section 1: Activity 3 (continued)

33. $f(x) = -x^2 - 4x - 12$

$= (-x^2 - 4x) - 12$

$= -1(x^2 + 4x) - 12$

$= -1[(x^2 + 4x + 4) - 4] - 12$

$= -1[(x+2)^2 - 4] - 12$

$= -(x+2)^2 + 4 - 12$

$= -(x+2)^2 - 8$

Therefore, the vertex is $(-2, -8)$ and the axis of symmetry is $x = -2$.

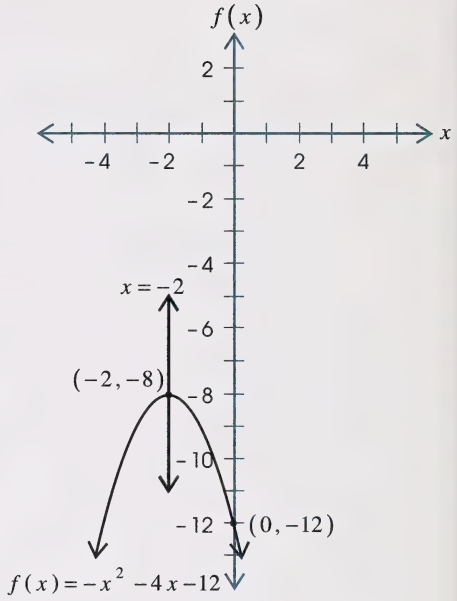
Because the graph opens downward from $(-2, -8)$, there are no x-intercepts.

To find the $f(x)$-intercept, let $x = 0$.

$\therefore f(x) = -x^2 - 4x - 12$

$f(0) = -(0)^2 - 4(0) - 12$

$= -12$



The $f(x)$-intercept is -12.

Because the graph opens downward and $(-2, -8)$ is a maximum point, the range is $f(x) \leq -8$.

36. The function $y = x^2 + 6x + 2$ has a minimum value because its graph opens upward $(a > 0)$.

The minimum occurs at the vertex; therefore, find the vertex by completing the square.

$$\begin{aligned} y &= x^2 + 6x + 2 \\ &= \left(x^2 + 6x\right) + 2 \\ &= \left(x^2 + 6x + 9\right) - 9 + 2 \\ &= (x+3)^2 - 7 \end{aligned}$$

The vertex is $(-3, -7)$.

Therefore, the minimum value of the function is $y = -7$.

38. The function $f(x) = -x^2 + 8x$ has a maximum value because its graph opens downward, $(a < 0)$.

Find the vertex by completing the square.

$$\begin{aligned} f(x) &= -x^2 + 8x \\ &= -1\left(x^2 - 8x\right) \\ &= -1\left[\left(x^2 - 8x + 16\right) - 16\right] \\ &= -1\left[(x-4)^2 - 16\right] \\ &= -(x-4)^2 + 16 \end{aligned}$$

The vertex is $(4, 16)$.

Therefore, the maximum value of the function is $y = 16$.

Section 1: Activity 3 (continued)

47. The function $y = -3x^2 + 18x - 28$ has a maximum value because its graph opens downward.

Find the vertex by completing the square.

$$\begin{aligned} y &= -3x^2 + 18x - 28 \\ &= \left(-3x^2 + 18x\right) - 28 \\ &= -3\left(x^2 - 6x\right) - 28 \\ &= -3\left[\left(x^2 - 6x + 9\right) - 9\right] - 28 \\ &= -3\left[(x-3)^2 - 9\right] - 28 \\ &= -3(x-3)^2 + 27 - 28 \\ &= -3(x-3)^2 - 1 \end{aligned}$$

The vertex of the graph is $(3, -1)$.

Therefore, the maximum value, $y = -1$, occurs when $x = 3$.

49. The function $y = 10x^2 - 20x + 12$ has a minimum value because the graph opens upward.

Find the vertex by completing the square.

$$\begin{aligned} y &= 10x^2 - 20x + 12 \\ &= \left(10x^2 - 20x\right) + 12 \\ &= 10\left(x^2 - 2x\right) + 12 \\ &= 10\left[\left(x^2 - 2x + 1\right) - 1\right] + 12 \\ &= 10\left[(x-1)^2 - 1\right] + 12 \\ &= 10(x-1)^2 - 10 + 12 \\ &= 10(x-1)^2 + 2 \end{aligned}$$

The vertex is $(1, 2)$.

Therefore, the minimum value, $y = 2$, occurs when $x = 1$.

52. The function $y = x^2 + 3x + 1$ has a minimum value because its graph opens upward.

Find the vertex.

$$\begin{aligned} y &= x^2 + 3x + 1 \\ &= \left(x^2 + 3x\right) + 1 \\ &= \left(x^2 + 3x + \frac{9}{4}\right) - \frac{9}{4} + 1 \\ &= \left(x + \frac{3}{2}\right)^2 - \frac{9}{4} + \frac{4}{4} \\ &= \left(x + \frac{3}{2}\right)^2 - \frac{5}{4} \end{aligned}$$

The vertex is $\left(-\frac{3}{2}, -\frac{5}{4}\right)$.

Therefore, the minimum value, $y = -\frac{5}{4}$, occurs when $x = -\frac{3}{2}$.

54. The function $y = -\frac{1}{3}x^2 + 2x + 4$ has a maximum value because its graph opens downward.

$$\begin{aligned} y &= \left(-\frac{1}{3}x^2 + 2x\right) + 4 \\ &= -\frac{1}{3}\left(x^2 - 6x\right) + 4 \\ &= -\frac{1}{3}\left[\left(x^2 - 6x + 9\right) - 9\right] + 4 \\ &= -\frac{1}{3}\left[(x-3)^2 - 9\right] + 4 \\ &= -\frac{1}{3}(x-3)^2 + 3 + 4 \\ &= -\frac{1}{3}(x-3)^2 + 7 \end{aligned}$$

The vertex is $(3, 7)$.

Therefore, the maximum value, $y = 7$, occurs when $x = 3$.

Section 1: Activity 3 (continued)

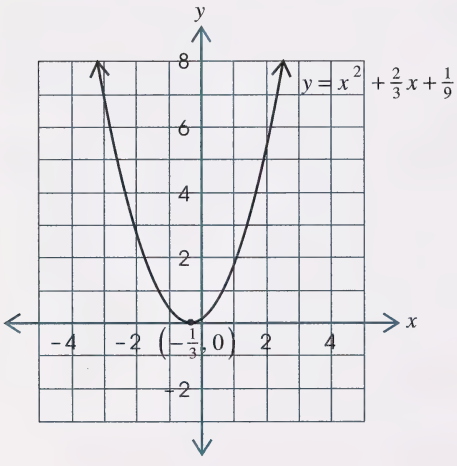
58. The function $y=-2x^2-0.8x-2$ has a maximum value because the graph opens downward.

$$\begin{aligned}
y&=\left(-2x^2-0.8x\right)-2\\
&=-2\left(x^2+0.4x\right)-2\\
&=-2\left[\left(x^2+0.4x+0.04\right)-0.04\right]-2\\
&=-2\left[(x+0.2)^2-0.04\right]-2\\
&=-2(x+0.2)^2+0.08-2\\
&=-2(x+0.2)^2-1.92
\end{aligned}$$

The vertex is $(-0.2,-1.92)$.

Therefore, the maximum value, $y=-1.92$, occurs when $x=-0.2$.

66.



$$\begin{aligned}
y&=x^2+\frac{2}{3}x+\frac{1}{9}\\
&=\left(x+\frac{1}{3}\right)^2
\end{aligned}$$

The vertex is $\left(-\frac{1}{3},0\right)$.

Therefore, the x-intercept is $-\frac{1}{3}$.

4. Textbook questions 70.a., 70.c., 71, 73, 74, 75, 80, 81, 82, 85, 87, 92, and 93 of "Applications and Problem Solving," pp. 132 to 134

70. a. To find the x-intercepts, let $y = 0$.

$\therefore\ y = (x+1)(x+3)$

$0 = (x+1)(x+3)$

$\therefore\ x+1 = 0 \quad \text{or} \quad x+3 = 0$

$x = -1 \qquad\qquad x = -3$

The x-intercepts are -3 and -1.

To find the y-intercept, let $x = 0$.

$\therefore\ y = (x+1)(x+3)$

$= (0+1)(0+3)$

$= (1)(3)$

$= 3$

The y-intercept is 3.

To find the vertex, complete the square.

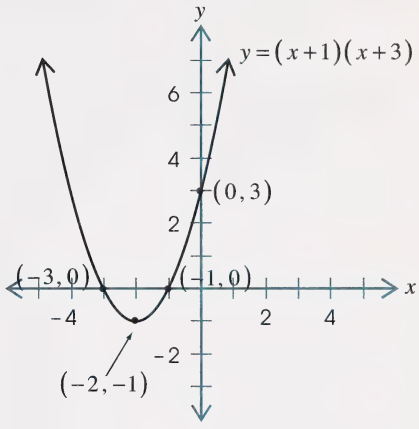
$y = (x+1)(x+3)$

$= x^2 + 4x + 3$

$= \left(x^2 + 4x + 4\right) - 4 + 3$

$= (x+2)^2 - 1$

The vertex is $(-2, -1)$.



Section 1: Activity 3 (continued)

c. To find the x-intercepts, let $f(x)=0$.

$$\therefore f(x)=(2x-3)^2$$
$$0=(2x-3)^2$$
$$0=2x-3$$
$$2x=3$$
$$x=\frac{3}{2}$$

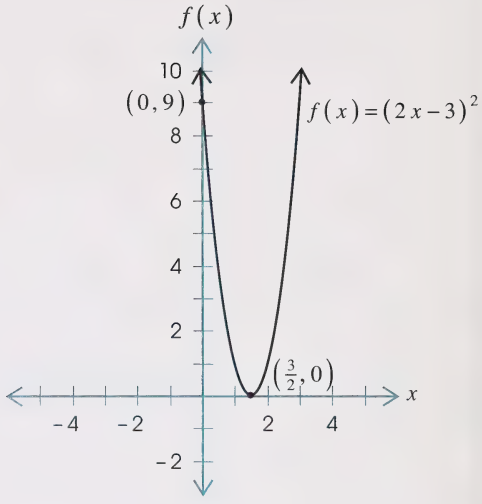
The x-intercept is $\frac{3}{2}$.

To find the $f(x)$-intercept, let $x=0$.

$$\therefore f(x)=(2x-3)^2$$
$$f(0)=[2(0)-3]^2$$
$$=(-3)^2$$
$$=9$$

The $f(x)$-intercept is 9.

Because there is a single x-intercept, the vertex is $\left(\frac{3}{2},0\right)$.



71. a. $y=x^2-8x+35$

b. To find the original number, x, that gives the least result, y, complete the square.

$$y=x^2-8x+35$$
$$=\left(x^2-8x\right)+35$$
$$=\left(x^2-8x+16\right)-16+35$$
$$=(x-4)^2+19$$

The least result, 19, occurs when the original number is 4.

73. Let x be one number; let $x+10$ be the other number; and let y be the product of the two numbers.

$$\begin{aligned}\therefore\ y &= x(x+10)\\ &= x^2+10x\end{aligned}$$

Complete the square.

$$\begin{aligned}y &= x^2+10x\\ &= \left(x^2+10x+25\right)-25\\ &= (x+5)^2-25\end{aligned}$$

The minimum product occurs when $x=-5$.

$$\begin{aligned}\therefore\ x+10 &= -5+10\\ &= 5\end{aligned}$$

Therefore, the two numbers that differ by 10 and yield a minimum product are -5 and 5.

74. Let x be one number; let $34-x$ be the other number; and let y be the product of the two numbers.

$$\begin{aligned}\therefore\ y &= x(34-x)\\ &= 34x-x^2\end{aligned}$$

Complete the square.

$$\begin{aligned}y &= 34x-x^2\\ &= -x^2+34x\\ &= -1\left(x^2-34x\right)\\ &= -1\left[\left(x^2-34x+289\right)-289\right]\\ &= -1\left[(x-17)^2-289\right]\\ &= -(x-17)^2+289\end{aligned}$$

The maximum product occurs when $x=17$.

$$\begin{aligned}\therefore\ 34-x &= 34-17\\ &= 17\end{aligned}$$

Therefore, the two numbers are 17 and 17.

Section 1: Activity 3 (continued)

75. Let x be one number; let $34-x$ the other number; and let y be the sum of their squares.

$\therefore\ y=x^2+(34-x)^2$

Complete the square.

$$\begin{aligned}
y &= x^2+(34-x)^2\\
&= x^2+1156-68x+x^2\\
&= 2x^2-68x+1156\\
&= \left(2x^2-68x\right)+1156\\
&= 2\left(x^2-34x\right)+1156\\
&= 2\left[\left(x^2-34x+289\right)-289\right]+1156\\
&= 2\left[(x-17)^2-289\right]+1156\\
&= 2(x-17)^2-578+1156\\
&= 2(x-17)^2+578
\end{aligned}$$

The minimum occurs when $x=17$.

$$\begin{aligned}
\therefore\ 34-x &= 34-17\\
&= 17
\end{aligned}$$

Therefore, the two numbers are 17 and 17.

80. a. The width of the arch at the base is the distance between the d-intercepts.

Let $h(d)=0$.

$$\therefore\ h(d)=-0.0425d^2+3.57d$$
$$0=d(-0.0425d+3.57)$$

$$\therefore\ d=0 \quad \text{or} \quad -0.0425d+3.57=0$$
$$0.0425d=3.57$$
$$d=\frac{3.57}{0.0425}$$
$$=84$$

Therefore, the width of the arch at the base is 84 m.

b. The maximum height will occur halfway between the d-intercepts, or at 42 m.

$$\therefore \text{ maximum height} = h(42)$$
$$=-0.0425(42)^2+3.57(42)$$
$$\doteq 75$$

The maximum height is approximately 75 m.

c. A horizontal distance 10 m from the vertex occurs at $d=42-10=32$ m.

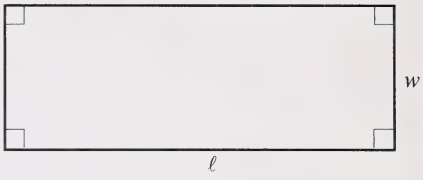
$$\therefore\ h(d)=-0.0425d^2+3.57d$$
$$h(32)=-0.0425(32)^2+3.57(32)$$
$$\doteq 71$$

At a horizontal distance of 10 m from the vertex, the height of the arch is approximately 71 m.

Section 1: Activity 3 (continued)

81. a. Let w be the width; let ℓ be the length; and let A be the area.

$\therefore A = \ell w$ (1)



Because the perimeter is 400 m, solve for ℓ.

$$2\ell + 2w = 400$$
$$\ell + w = 200$$
$$\ell = 200 - w \quad (2)$$

Substitute (2) into (1).

$$A = \ell w$$
$$= (200 - w)w$$
$$= 200w - w^2$$

Complete the square.

$$A = 200w - w^2$$
$$= -w^2 + 200w$$
$$= -1(w^2 - 200w)$$
$$= -1\left[(w^2 - 200w + 10\,000) - 10\,000\right]$$
$$= -1\left[(w - 100)^2 - 10\,000\right]$$
$$= -(w - 100)^2 + 10\,000$$

The maximum area occurs when $w = 100$.

$$\therefore \ell = 200 - w$$
$$= 200 - 100$$
$$= 100$$

A square 100 m on a side yields the maximum area.

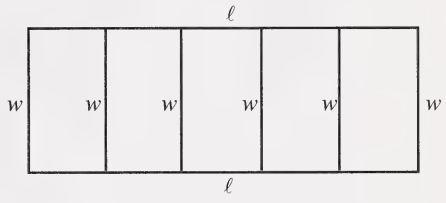
b. $A = \ell w$

$= 100(100)$

$= 10\,000$

The maximum area is $10\,000\ \text{m}^2$.

82. Let ℓ be the length, and let w be the width.



$\therefore\ 2\ell + 6w = 120$

$\ell + 3w = 60$

$\ell = 60 - 3w$

Let A be the area enclosed.

$\therefore\ A = \ell w$

$= (60 - 3w)w$

$= 60w - 3w^2$

Complete the square.

$A = -3w^2 + 60w$

$= -3\left(w^2 - 20w\right)$

$= -3\left[\left(w^2 - 20w + 100\right) - 100\right]$

$= -3\left[(w-10)^2 - 100\right]$

$= -3(w-10)^2 + 300$

The maximum area occurs when $w = 10$.

$\therefore\ \ell = 60 - 3w$

$= 60 - 3(10)$

$= 30$

The overall dimensions of the enclosure are 10 m × 30 m.

Section 1: Activity 3 (continued)

85. Let x be the number of \$2 increases in price.

Therefore, the price of one shirt is $20+2x$ and the number of shirts sold is $1200-60x$.

Let R represent the revenue.

$$\begin{aligned}\therefore R &= (\text{number of shirts sold})\times(\text{price per shirt})\\ &= (1200-60x)(20+2x)\\ &= 24\,000+2400x-1200x-120x^2\\ &= -120x^2+1200x+24\,000\end{aligned}$$

Complete the square.

$$\begin{aligned}R &= -120x^2+1200x+24\,000\\ &= \left(-120x^2+1200x\right)+24\,000\\ &= -120\left(x^2-10x\right)+24\,000\\ &= -120\left[\left(x^2-10x+25\right)-25\right]+24\,000\\ &= -120\left[(x-5)^2-25\right]+24\,000\\ &= -120(x-5)^2+3000+24\,000\\ &= -120(x-5)^2+27\,000\end{aligned}$$

The number of \$2 increases is 5.

Therefore, shirts should be sold at $20+2(5)=\$30$ in order to maximize the revenue.

87. a. Complete the square to determine the maximum height.

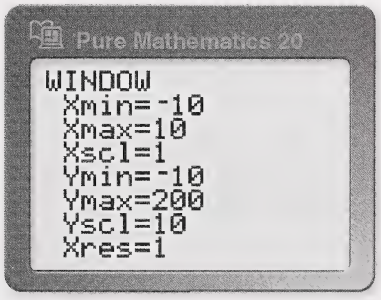
$$\begin{aligned}
h(t) &= -4.9t^2 + 24.5t + 146 \\
&= \left(-4.9t^2 + 24.5t\right) + 146 \\
&= -4.9\left(t^2 - 5t\right) + 146 \\
&= -4.9\left[\left(t^2 - 5t + 6.25\right) - 6.25\right] + 146 \\
&= -4.9\left[(t-2.5)^2 - 6.25\right] + 146 \\
&= -4.9(t-2.5)^2 + 30.625 + 146 \\
&= -4.9(t-2.5)^2 + 176.625
\end{aligned}$$

Therefore, the maximum height of the object would be approximately 176.6 m.

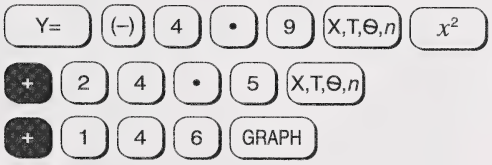
b. When the object strikes the ground, $h(t) = 0$.

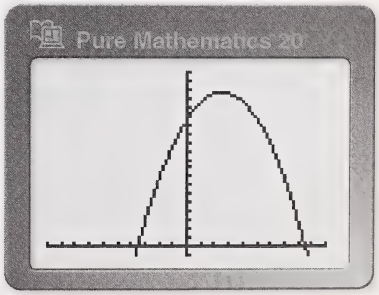
Graph $h(t) = -4.9t^2 + 24.5t + 146$ on your graphing calculator and use the Zero feature to determine when $h(t) = 0$.

Set the window to the following settings.



Graph the equation.

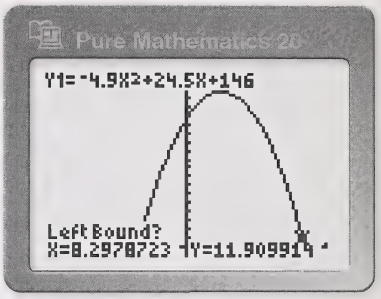




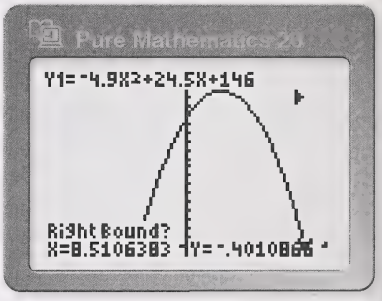
Section 1: Activity 3 (continued)

Access the Zero feature by pressing [2nd] [CALC] [2].

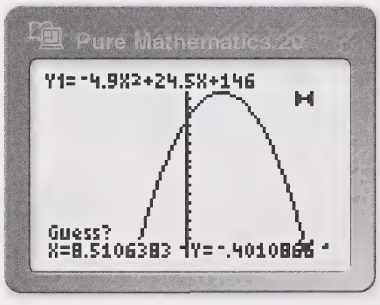
Use the arrow keys to position the cursor to the left of the zero. Then press [ENTER] to store the left bound.

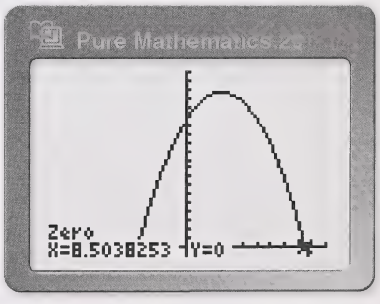


Use the arrow key to position the cursor to the right of the zero. Press [ENTER] to store the right bound.



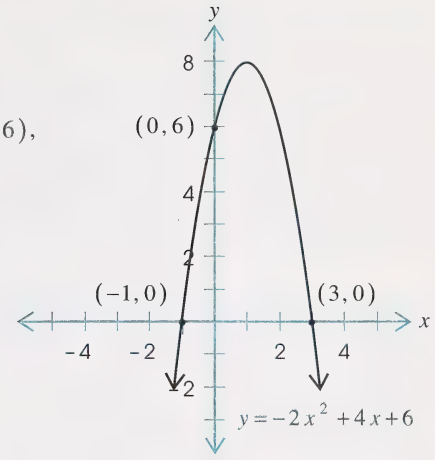
To enter your guess, position the cursor between the left bound and the right bound and press [ENTER].





The zero, correct to the nearest tenth, is 8.5. Therefore, the object is in the air for approximately 8.5 s.

92. Because the x-intercepts are -1 and 3, the equation of the parabola is of the form $y = a(x+1)(x-3)$.



To determine the value of a, substitute the y-intercept, $(0, 6)$, into the equation.

$$y = a(x+1)(x-3)$$
$$6 = a(0+1)(0-3)$$
$$6 = a(1)(-3)$$
$$6 = -3a$$
$$a = -2$$

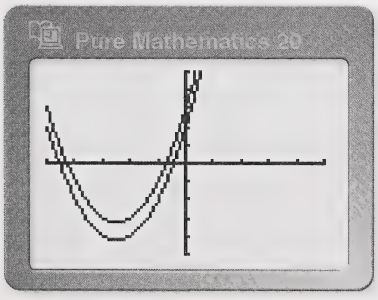
$$\therefore y = -2(x+1)(x-3)$$
$$= -2\left(x^2 - 3x + x - 3\right)$$
$$= -2\left(x^2 - 2x - 3\right)$$
$$= -2x^2 + 4x + 6$$

93. To obtain the points of intersection, if any, solve the system of equations.

$$y = x^2 + 5x + 2 \quad (1)$$
$$y = x^2 + 5x + 3 \quad (2)$$
$$0 = 0 + 0 - 1 \quad (1) - (2)$$
$$0 = -1$$

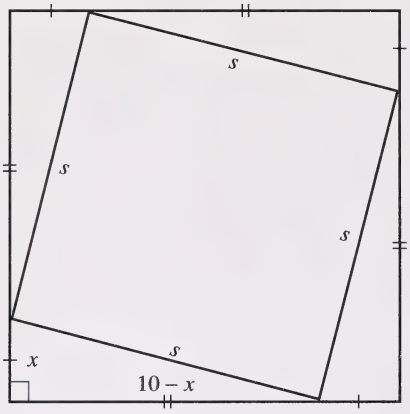
Impossible!

Therefore, there are no common points and the graphs do not intersect.



Section 1: Activity 3 (continued)

5. Textbook question 88 of "Applications and Problem Solving," p. 134



Let x be the shorter leg of each right triangle; let $10-x$ be the longer leg of each right triangle; and let s be the length of each side of the inner square.

Using the Pythogorean Theorem, $s^2 = x^2 + (10-x)^2$

$\therefore$ Area of inner square $= s^2$

$$= x^2 + (10-x)^2$$
$$= x^2 + 100 - 20x + x^2$$
$$= 2x^2 - 20x + 100$$
$$= 2\left(x^2 - 10x\right) + 100 \quad \longleftarrow \text{Complete the square.}$$
$$= 2\left[\left(x^2 - 10x + 25\right) - 25\right] + 100$$
$$= 2\left[(x-5)^2 - 25\right] + 100$$
$$= 2(x-5)^2 - 50 + 100$$
$$= 2(x-5)^2 + 50$$

The minimum area of the inner square occurs when $x = 5$ cm. The minimum area is 50 cm^2.

Section 1: Follow-up Activities

Extra Help

1. Textbook questions 1 and 2 of Investigation 1, "Making Squares," p. 125

1. a. **b.**

2. a. $x^2+4x+4=(x+2)^2$ **b.** $x^2+6x+9=(x+3)^2$

2. Textbook questions 1 to 6 of Investigation 2, "Completing the Square," p. 125

1. To complete the square, 16 one-tiles must be added.

2. $x^2+8x+16=(x+4)^2$

3. a. To complete the square, 25 one-tiles must be added.

$\therefore\ x^2+10x+25=(x+5)^2$

b. To complete the square, 36 one-tiles must be added.

$\therefore\ x^2+12x+36=(x+6)^2$

4. The number of one-tiles added to each display is the square of one-half the number of x-tiles.

5. a. $\left(\frac{14}{2}\right)^2=7^2$
$=49$

b. $\left(\frac{16}{2}\right)^2=8^2$
$=64$

c. $\left(\frac{20}{2}\right)^2=10^2$
$=100$

d. $\left(\frac{30}{2}\right)^2=15^2$
$=225$

6. a. $x^2+14x+49=(x+7)^2$ **b.** $x^2+16x+64=(x+8)^2$

c. $x^2+20x+100=(x+10)^2$ **d.** $x^2+30x+225=(x+15)^2$

Enrichment

1. $a = -4$

$\therefore \Delta^2 y = 2a$

$= 2(-4)$

$= -8$

x	0	1	2	3	4
y	1	−4	−17	−38	−67

$\Delta y \longrightarrow$ −5 −13 −21 −29

$\Delta^2 y \longrightarrow$ −8 −8 −8

2.

x	0	1	2
y	10	3	12

$\Delta y \longrightarrow$ −7 9

$\Delta^2 y \longrightarrow$ 16

Because $\Delta^2 y = 16$,

$2a = 16$

$a = 8$

Also, because $(0, 10)$ satisfies the equation, $c = 10$.

$\therefore y = 8x^2 + bx + 10$

To determine b, substitute $(1, 3)$ into the equation.

$y = 8x^2 + bx + 10$

$3 = 8(1)^2 + b(1) + 10$

$3 = 8 + b + 10$

$3 = b + 18$

$b = -15$

Therefore, the equation of the quadratic function is $y = 8x^2 - 15x + 10$.

3. Textbook questions 1 to 6 of Investigation 1, "Falling Objects," p. 140

1.

Elapsed Time (s)	Total Distance Fallen (m)	Instantaneous Speed (m/s)
5	125	50
6	180	60

2. $f(t)=5t^2$

3. If $t=7\text{s}$, then

$$f(t)=5t^2$$
$$f(7)=5(7)^2$$
$$=245\text{ m}$$

If $t=8\text{s}$, then

$$f(t)=5t^2$$
$$f(8)=5(8)^2$$
$$=320\text{ m}$$

4. The speed increases 10 m/s every second.

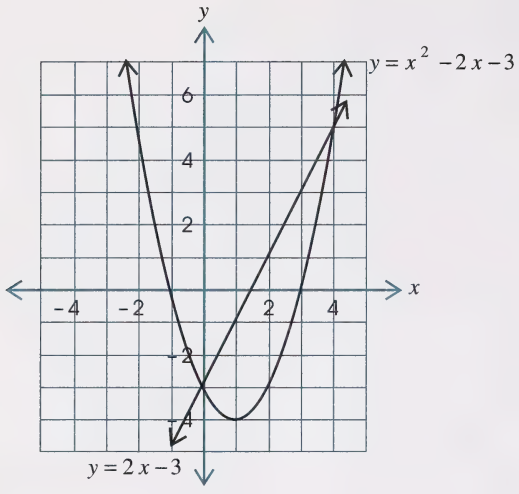
Therefore, $g=10\text{ m/s}^2$.

5. The coefficient of t^2 is half the value of g.

6. Answers may vary. Factors that affect terminal velocity are mass, density, shape, and size. Different objects reach their terminal velocities at different times. For example, a feather reaches its terminal velocity almost immediately after it is dropped.

Section 2: Activity 1

1. a. Textbook question "Explore: Interpret the Graphs," p. 152



To find the x-intercepts, let $y = 0$.

$\therefore\ y = 2x - 3$

$0 = 2x - 3$

$2x = 3$

$x = \frac{3}{2}$ or 1.5

$\therefore\ y = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$\therefore\ x - 3 = 0$ or $x + 1 = 0$

$x = 3$ $\quad$ $x = -1$

To find the y-intercepts, let $x = 0$.

$\therefore\ y = 2x - 3$

$= 2(0) - 3$

$= -3$

$\therefore\ y = x^2 - 2x - 3$

$= 0^2 - 2(0) - 3$

$= -3$

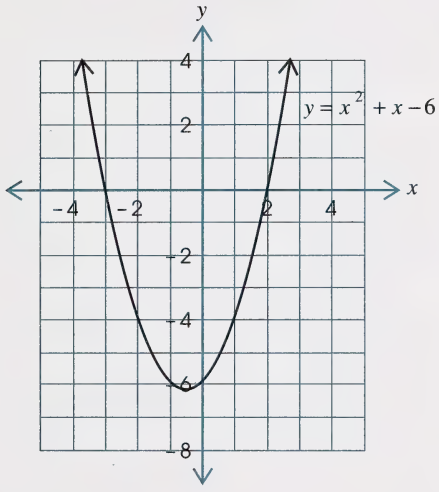
b. Textbook questions 1 to 4 of "Inquire," p. 152

1. The solution to $2x-3=0$ is $x=1.5$. This is the x-intercept of the graph of $y=2x-3$.

Remember: To find an x-intercept, replace y with 0 and solve the resulting equation.

2. The solutions to $x^2-2x-3=0$ are $x=-1$ and $x=3$. These are the x-intercepts of the graph of $y=x^2-2x-3$.

3. a.



b. The x-intercepts, from the graph, are -3 and 2.

c. Use LS/RS tables to verify that the x-intercepts satisfy $x^2+x-6=0$.

Check

For $x=-3$,

LS	RS
x^2+x-6	0
$=(-3)^2+(-3)-6$	
$=9-3-6$	
$=0$	

LS = RS

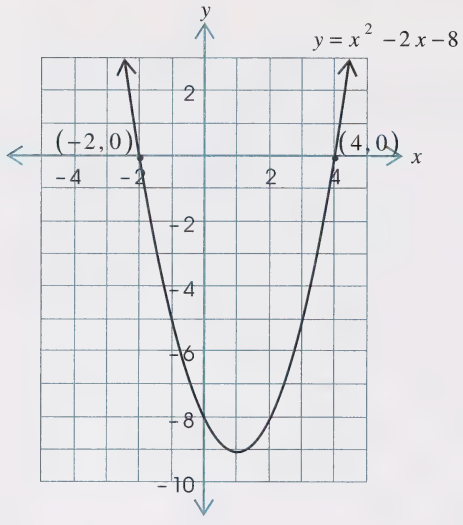
For $x=2$,

LS	RS
x^2+x-6	0
$=2^2+2-6$	
$=4+2-6$	
$=0$	

LS = RS

Section 2: Activity 1 (continued)

4. a. Graph the related quadratic function, $y = x^2 - 2x - 8$.



From the graph, the x-intercepts are -2 and 4.

The x-intercepts of the graph of $y = x^2 - 2x - 8$ are the roots of the equation $x^2 - 2x - 8 = 0$.

Check

For $x = -2$,

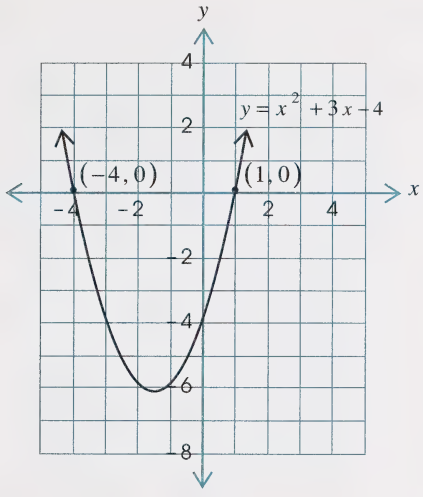
	LS	RS
$x^2 - 2x - 8$		0
$= (-2)^2 - 2(-2) - 8$		
$= 4 + 4 - 8$		
$= 0$		
	LS =	RS

For $x = 4$,

	LS	RS
$x^2 - 2x - 8$		0
$= 4^2 - 2(4) - 8$		
$= 16 - 8 - 8$		
$= 0$		
	LS =	RS

Therefore, the solution set of $x^2 - 2x - 8 = 0$ is $\{-2, 4\}$.

b. Graph the related quadratic function, $y = x^2 + 3x - 4$.



From the graph, the x-intercepts are -4 and 1.

The x-intercepts of the graph of $y = x^2 + 3x - 4$ are the roots of the equation $x^2 + 3x - 4 = 0$.

Check

For $x = -4$,

LS	RS
$x^2 + 3x - 4$	0
$= (-4)^2 + 3(-4) - 4$	
$= 16 - 12 - 4$	
$= 0$	

LS = RS

For $x = 1$,

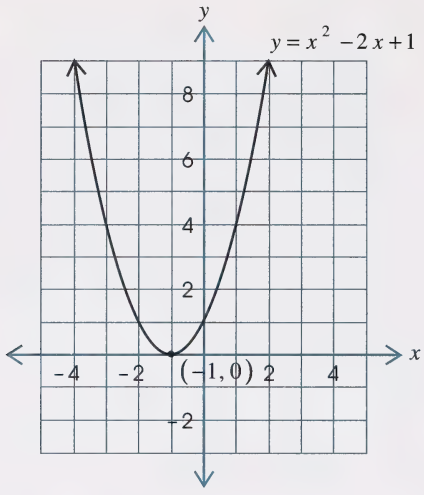
LS	RS
$x^2 + 3x - 4$	0
$= 1^2 + 3(1) - 4$	
$= 1 + 3 - 4$	
$= 0$	

LS = RS

Therefore, the solution set of $x^2 + 3x - 4 = 0$ is $\{-4, 1\}$.

Section 2: Activity 1 (continued)

c. Graph the related function, $y = x^2 + 2x + 1$.



From the graph, the x-intercept is -1.

The x-intercept of the graph of $y = x^2 + 2x + 1$ is the root of the equation $x^2 + 2x + 1 = 0$.

Check

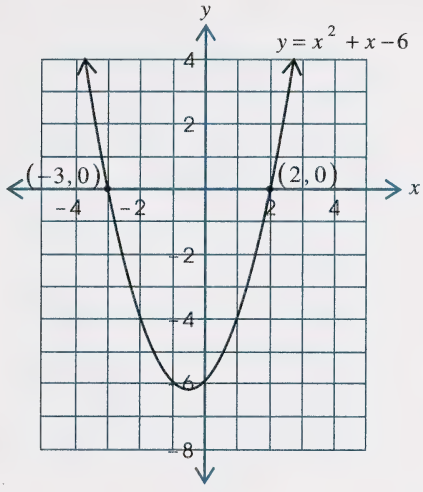
LS	RS
$x^2 + 2x + 1$	0
$= (-1)^2 + 2(-1) + 1$	
$= 1 - 2 + 1$	
$= 0$	

LS = RS

Therefore, the solution of $x^2 + 2x + 1 = 0$ is $x = -1$.

2. Textbook questions 1, 7, 8, 22, and 30 of "Practice," p. 155

1. Graph the related function, $y = x^2 + x - 6$.



The roots are -3 and 2.

Check

For $x = -3$,

LS	RS
$x^2 + x - 6$	0
$= (-3)^2 + (-3) - 6$	
$= 9 - 3 - 6$	
$= 0$	

LS = RS

For $x = 2$,

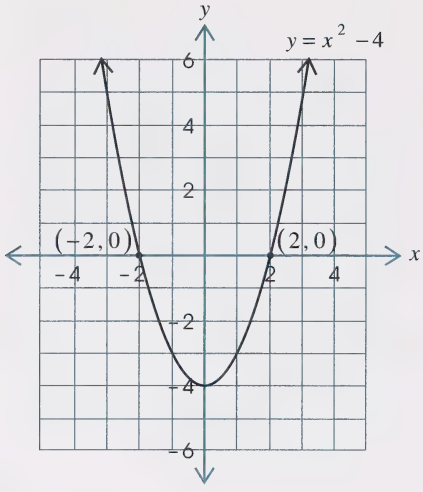
LS	RS
$x^2 + x - 6$	0
$= 2^2 + 2 - 6$	
$= 4 + 2 - 6$	
$= 0$	

LS = RS

Therefore, the roots of equation $x^2 + x - 6 = 0$ are -3 and 2.

Section 2: Activity 1 (continued)

7. Graph the related function, $y = x^2 - 4$.



The roots are -2 and 2.

Check

For $x = -2$,

LS	RS
$x^2 - 4$	0
$= (-2)^2 - 4$	
$= 4 - 4$	
$= 0$	

LS = RS

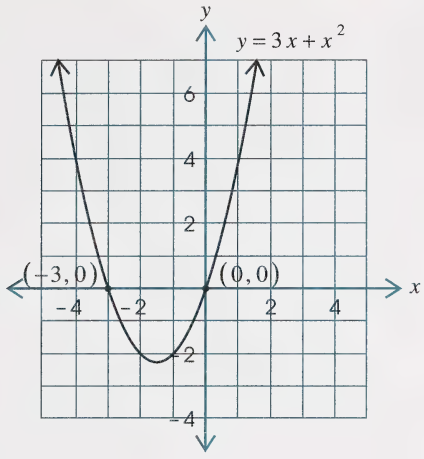
For $x = 2$,

LS	RS
$x^2 - 4$	0
$= 2^2 - 4$	
$= 4 - 4$	
$= 0$	

LS = RS

Therefore, the roots of equation $x^2 - 4 = 0$ are -2 and 2.

8. Graph the related function, $y = 3x + x^2$.



The roots are -3 and 0.

Check

For $x = -3$,

LS	RS
$3x + x^2$	0
$= 3(-3) + (-3)^2$	
$= -9 + 9$	
$= 0$	

LS = RS

For $x = 0$,

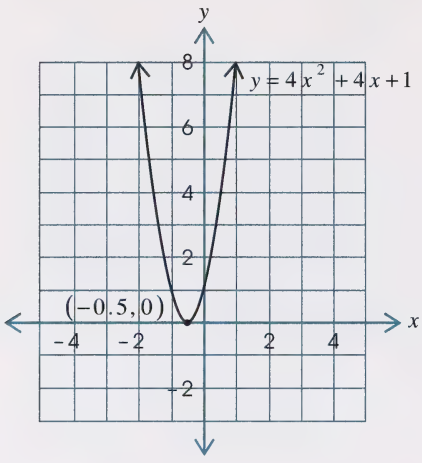
LS	RS
$3x + x^2$	0
$= 3(0) + 0^2$	
$= 0 + 0$	
$= 0$	

LS = RS

Therefore, the roots of equation $3x + x^2 = 0$ are -3 and 0.

Section 2: Activity 1 (continued)

22. Graph the related function, $y = 4x^2 + 4x + 1$.



The root is -0.5.

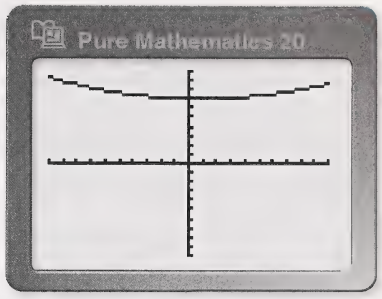
Check

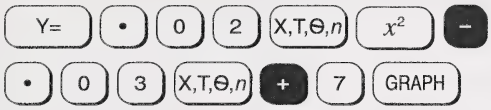
LS	RS
$4x^2 + 4x + 1$	0
$= 4(-0.5)^2 + 4(-0.5) + 1$	
$= 4(0.25) - 2 + 1$	
$= 1 - 2 + 1$	
$= 0$	

LS = RS

Therefore, the root of equation $0 = 4x^2 + 4x + 1$ is -0.5.

30. Graph the related function on your graphing calculator.





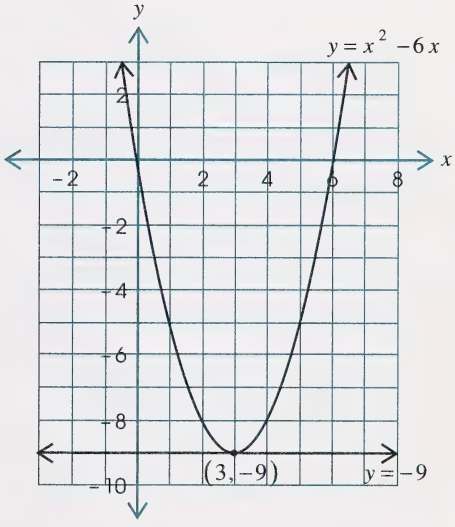
The graph does not cross the x-axis. Therefore, there are no real roots of equation $0.02x^2 - 0.03x + 7 = 0$.

3. a. Textbook question 15 of "Practice," p. 155

15. Graph the following system of equations:

$y = x^2 - 6x$ (1)

$y = -9$ (2)



Find the points where the graphs intersect each other.

The intersection point is $(3, -9)$.

The root of the equation is the x-coordinate of the intersection point.

Check

LS	RS
$x^2 - 6x$	-9
$= 3^2 - 6(3)$	
$= 9 - 18$	
$= -9$	

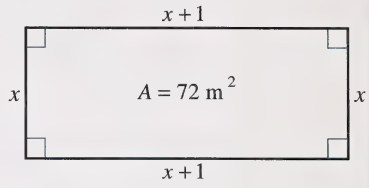
LS = RS

Therefore, the root of equation $x^2 - 6x = -9$ is 3.

Section 2: Activity 1 (continued)

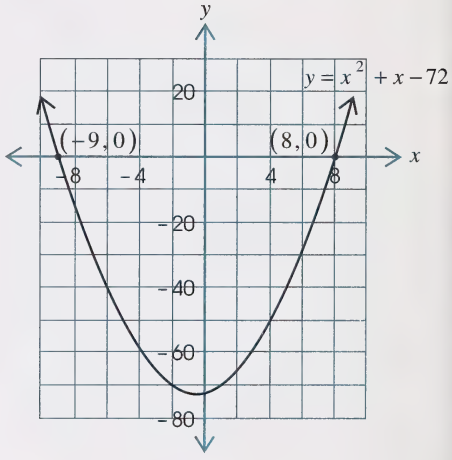
b. Textbook questions 41, 43, 45, 48, and 58 of "Applications and Problem Solving," p. 156

41. Let x be the width (in metres), and let $x+1$ be the length (in metres).



$$\therefore A = \text{length} \times \text{width}$$
$$72 = x(x+1)$$
$$72 = x^2 + 2$$
$$0 = x^2 + x - 72$$
$$x^2 + x - 72 = 0$$

To solve the equation, graph the related quadratic function, $y = x^2 + x - 72$; then determine the x-intercepts of the graph.



The roots are -9 and 8. However, the root -9 is inadmissible because length must be positive; so, $x = 8$.

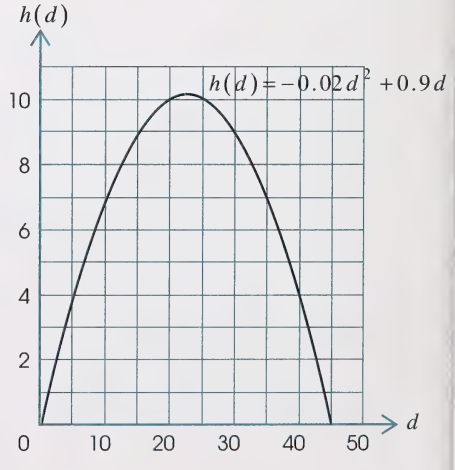
Therefore, the width is 8 m, and the length is $8+1=9$ m.

Note: $8\text{ m} \times 9\text{ m} = 72\text{ m}^2$, the area of the rectangle.

43. $h(d) = -0.02d^2 + 0.9d$

When the ball strikes the ground, $h(d) = 0$.

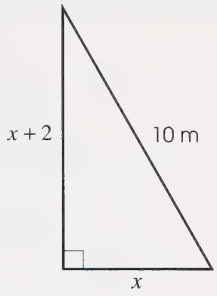
Determine the d-intercepts of the graph of $h(d) = -0.02d^2 + 0.9d$.



The d-intercepts are 45 m apart. Therefore, the ball travels 45 m before it strikes the ground.

Note: $h(45) = -0.02(45)^2 + 0.9(45)$
$= 0$

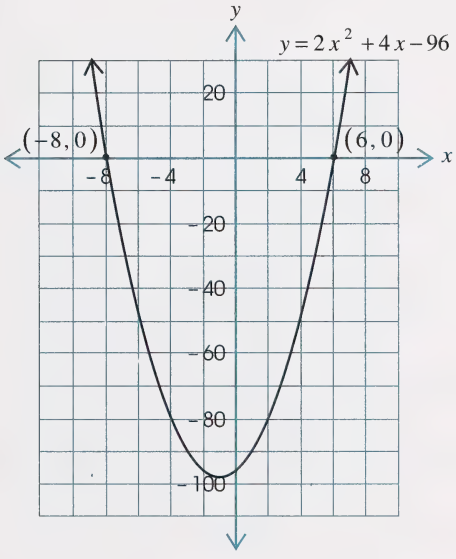
45. Let x be the length (in metres) of one leg of the right triangle, and let $x+2$ be the length (in metres) of the other leg.



Using the Pythagorean Theorem,

$$x^2+(x+2)^2=10^2$$
$$x^2+x^2+4x+4=100$$
$$2x^2+4x-96=0$$

Graph the related quadratic function, $y=2x^2+4x-96$; then locate the x-intercepts of the graph.



The x-intercepts of the graph are -8 and 6. Therefore, the roots of $2x^2+4x-96=0$ are -8 and 6. However, the root -8 is inadmissible because length must be positive.

Therefore, the legs are 6 m and $6+2=8$ m in length.

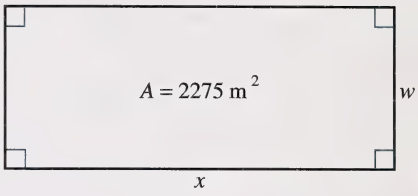
Note: The lengths of the legs satisfy the Pythagorean Theorem for a hypotenuse of 10.

LS	RS
6^2+8^2	10^2
$=48+64$	$=100$
$=100$	

LS = RS

Section 2: Activity 1 (continued)

48. Let x be the length (in metres), and let w be the width (in metres).

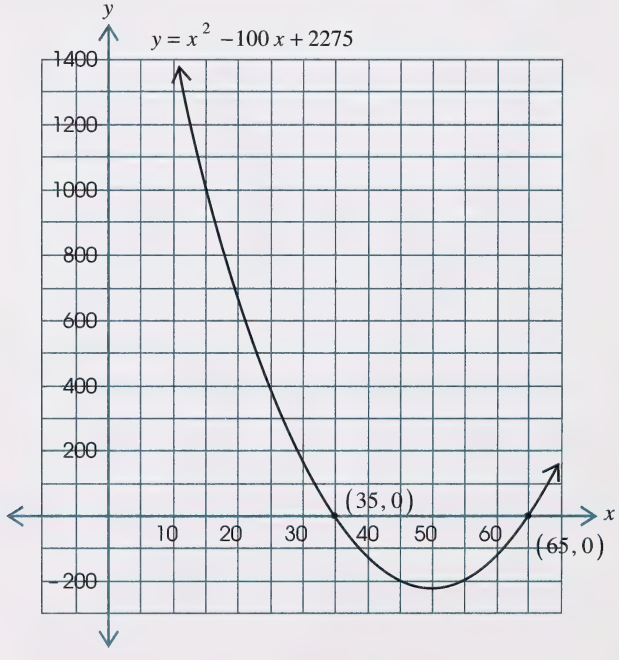


Because the perimeter is 200 m,

$$2x + 2w = 200$$
$$x + w = 100$$
$$w = 100 - x$$

$$\therefore A = \text{length} \times \text{width}$$
$$2275 = x(100 - x)$$
$$2275 = 100x - x^2$$
$$x^2 - 100x + 2275 = 0$$

Graph the related quadratic function, $y = x^2 - 100x + 2275$; then determine the x-intercepts.



The x-intercepts of the graph are 35 and 65. Therefore, the roots of $y = x^2 - 100x + 2275$ are 35 and 65.

If $x = 35$, then

$$\begin{aligned} w &= 100 - x \\ &= 100 - 35 \\ &= 65 \end{aligned}$$

If $x = 65$, then

$$\begin{aligned} w &= 100 - x \\ &= 100 - 65 \\ &= 35 \end{aligned}$$

Therefore, the dimensions of the field are $35\text{ m} \times 65\text{ m}$.

58. a. The equation $x^2 + 14x + c = 0$ has two equal real roots when the trinomial is a perfect square.

$$\begin{aligned} \therefore\ c &= \left(\frac{14}{2}\right)^2 \\ &= 49 \end{aligned}$$

b. When $c = 49$, the related quadratic function is

$$\begin{aligned} y &= x^2 + 14x + 49 \\ &= (x+7)^2 \end{aligned}$$

This is a parabola that opens upward from $(-7, 0)$. To cross the x-axis twice, the constant, c, would have to be less then 49.

Therefore, $x^2 + 14x + c = 0$ would have two real and distinct roots when $c < 49$.

c. If $c > 49$, the graph of $y = x^2 + 14x + c$ would not cross the x-axis, thus having no real roots.

4. Textbook questions 1 to 24 of "Mental Math: Simplifying and Evaluating Square Roots," p. 151

1. $\sqrt{8} = \sqrt{4 \times 2}$
$= 2\sqrt{2}$

2. $\sqrt{18} = \sqrt{9 \times 2}$
$= 3\sqrt{2}$

3. $\sqrt{15}$ cannot be simplified.

4. $\sqrt{12} = \sqrt{4 \times 3}$
$= 2\sqrt{3}$

5. $\sqrt{22}$ cannot be simplified.

6. $\sqrt{24} = \sqrt{4 \times 6}$
$= 2\sqrt{6}$

Section 2: Activity 1 (continued)

7. $\sqrt{27} = \sqrt{9 \times 3}$
$= 3\sqrt{3}$

8. $\sqrt{44} = \sqrt{4 \times 11}$
$= 2\sqrt{11}$

9. $\sqrt{45} = \sqrt{9 \times 5}$
$= 3\sqrt{5}$

10. $\sqrt{50} = \sqrt{25 \times 2}$
$= 5\sqrt{2}$

11. $\sqrt{63} = \sqrt{9 \times 7}$
$= 3\sqrt{7}$

12. $\sqrt{80} = \sqrt{16 \times 5}$
$= 4\sqrt{5}$

13. $\sqrt{49} = 7$

14. $\sqrt{-9}$ is undefined.

15. $\sqrt{121} = 11$

16. $\sqrt{25 + 56} = \sqrt{81}$
$= 9$

17. $\sqrt{36 - 20} = \sqrt{16}$
$= 4$

18. $\sqrt{64 - 28} = \sqrt{36}$
$= 6$

19. $\sqrt{16 + 4(12)} = \sqrt{16 + 48}$
$= \sqrt{64}$
$= 8$

20. $\sqrt{10^2 - 36} = \sqrt{100 - 36}$
$= \sqrt{64}$
$= 8$

21. $\sqrt{7^2 + 32} = \sqrt{49 + 32}$
$= \sqrt{81}$
$= 9$

22. $\sqrt{3^2 - 4(2)(-2)} = \sqrt{9 + 16}$
$= \sqrt{25}$
$= 5$

23. $\sqrt{8^2 - 4(3)(-3)} = \sqrt{64 + 36}$
$= \sqrt{100}$
$= 10$

24. $\sqrt{7^2 - 4(2)(3)} = \sqrt{49 - 24}$
$= \sqrt{25}$
$= 5$

Section 2: Activity 2

1. a. Textbook questions a. to c. of "Explore: Solve an Equation," p. 157

a. Let $x + 12$ be the length.

b. Area = length × width
$= (x + 12)x$

c. $28 = (x + 12)x$
$28 = x^2 + 12x$

b. Textbook questions 1 to 4 of "Inquire," p. 157

1. $x^2+12x-28=0$

2. $(x+14)(x-2)=0$

3. a. $x+14=0$ or $x-2=0$

$x=-14$ $x=2$

b. The width cannot be negative; therefore, the width is 2 m and the length is $2+12=14$ m.

4. a. $x^2+5x+6=0$

$(x+2)(x+3)=0$

$\therefore x+2=0$ or $x+3=0$

$x=-2$ $x=-3$

The solutions are -2 and -3.

b. $x^2-3x-4=0$

$(x-4)(x+1)=0$

$\therefore x-4=0$ or $x+1=0$

$x=4$ $x=-1$

The solutions are 4 and -1.

c. $x^2+2=3x$

$x^2-3x+2=0$

$(x-2)(x-1)=0$

$\therefore x-2=0$ or $x-1=0$

$x=2$ $x=1$

The solutions are 2 and 1.

2. Textbook questions 6, 7, 12, 13, 25, 26, 32, 37, 44, 65, and 66 of "Practice," pp. 160 and 161

6. $(3x+4)(2x-1)=0$

$\therefore 3x+4=0$ or $2x-1=0$

$3x=-4$ $2x=1$

$x=-\frac{4}{3}$ $x=\frac{1}{2}$

The solutions are $-\frac{4}{3}$ and $\frac{1}{2}$.

Section 2: Activity 2 (continued)

7. $x(x+9)=0$

$\therefore\ x=0 \quad \text{or} \quad x+9=0$

$x=-9$

The solutions are 0 and -9.

12. $(x+1)^2=4$

$x^2+2x+1=4$

$x^2+2x-3=0$

13. $4m^2=3m$

$4m^2-3m=0$

25. $0=p^2+2p-35$

$0=(p+7)(p-5)$

$\therefore\ p+7=0 \quad \text{or} \quad p-5=0$

$p=-7 \qquad\qquad p=5$

Check

For $p=-7$,

LS	RS
0	$p^2+2p-35$
	$=(-7)^2+2(-7)-35$
	$=49-14-35$
	$=0$

LS = RS

For $p=5$,

LS	RS
0	$p^2+2p-35$
	$=5^2+2(5)-35$
	$=25+10-35$
	$=0$

LS = RS

The solutions are -7 and 5.

26. $m^2 - 7m = 18$

$m^2 - 7m - 18 = 0$

$(m-9)(m+2) = 0$

$\therefore\ m-9=0 \quad \text{or} \quad m+2=0$

$m=9 \qquad\qquad m=-2$

Check

For $m=9$,

LS	RS
$m^2 - 7m$	18
$=9^2 - 7(9)$	
$=81-63$	
$=18$	
LS =	RS

For $m=-2$,

LS	RS
$m^2 - 7m$	18
$=(-2)^2 - 7(-2)$	
$=4+14$	
$=18$	
LS =	RS

The solutions are 9 and -2.

32. $10y^2 - 16y = -6$

$10y^2 - 16y + 6 = 0$

$2\left(5y^2 - 8y + 3\right) = 0$

$2(5y-3)(y-1) = 0$

$\therefore\ 5y-3=0 \quad \text{or} \quad y-1=0$

$y=\frac{3}{5} \qquad\qquad y=1$

Section 2: Activity 2 (continued)

Check

For $y = \frac{3}{5}$,

LS	RS
$10y^2 - 16y$	-6
$= 10\left(\frac{3}{5}\right)^2 - 16\left(\frac{3}{5}\right)$	
$= \cancel{10}^{2}\left(\frac{9}{\cancel{25}_{5}}\right) - \frac{48}{5}$	
$= \frac{18}{5} - \frac{48}{5}$	
$= -\frac{30}{5}$	
$= -6$	

LS = RS

For $y = 1$,

LS	RS
$10y^2 - 16y$	-6
$= 10(1)^2 - 16(1)$	
$= 10 - 16$	
$= -6$	

LS = RS

The solutions are $\frac{3}{5}$ and 1.

37. $5t^2 - 20t = 0$

$5t(t-4) = 0$

$\therefore\ 5t = 0$ or $t - 4 = 0$

$t = 0$ $t = 4$

The solutions are 0 and 4.

44. $(x-6)^2 - 8x = 0$

$x^2 - 12x + 36 - 8x = 0$

$x^2 - 20x + 36 = 0$

$(x-2)(x-18) = 0$

$\therefore\ x - 2 = 0$ or $x - 18 = 0$

$x = 2$ $x = 18$

The solutions are 2 and 18.

65. $k^2+(k+1)^2+(k+2)^2=29$

$k^2+k^2+2k+1+k^2+4k+4=29$

$3k^2+6k+5=29$

$3k^2+6k-24=0$

$3\left(k^2+2k-8\right)=0$

$3(k+4)(k-2)=0$

$\therefore\ k+4=0$ or $k-2=0$

$k=-4$ $k=2$

The solutions are -4 and 2.

66. $(2z+1)(2z-1)+12=4(z+5)-10$

$4z^2-1+12=4z+20-10$

$4z^2+11=4z+10$

$4z^2-4z+1=0$

$(2z-1)(2z-1)=0$

$\therefore\ 2z-1=0$

$z=\frac{1}{2}$

The solution is $\frac{1}{2}$.

3.

LS	RS
$\frac{1}{x-1}+\frac{1}{x+1}$	$\frac{x}{x-1}$
$=\frac{1}{0-1}+\frac{1}{0+1}$	$=\frac{0}{0-1}$
$=\frac{1}{-1}+\frac{1}{1}$	$=\frac{0}{-1}$
$=-1+1$	$=0$
$=0$	

LS = RS

Therefore, 0 is a solution of the equation.

Section 2: Activity 2 (continued)

4. a. Textbook questions 50, 53, 59, 60, and 62 of "Practice," p. 161

50.

$$a = \frac{a-6}{a+6}$$

$$a(a+6) = a-6 \quad \longleftarrow \text{Multiply both sides by } a+6.$$

$$a^2 + 6a = a - 6$$

$$a^2 + 5a + 6 = 0$$

$$(a+2)(a+3) = 0$$

$$\therefore\ a+2 = 0 \quad \text{or} \quad a+3 = 0$$

$$a = -2 \qquad\qquad a = -3$$

Check

For $a = -2$,

LS	RS
a	$\frac{a-6}{a+6}$
$=-2$	$=\frac{-2-6}{-2+6}$
	$=\frac{-8}{4}$
	$=-2$

LS = RS

For $a = -3$,

LS	RS
a	$\frac{a-6}{a+6}$
$=-3$	$=\frac{-3-6}{-3+6}$
	$=\frac{-9}{3}$
	$=-3$

LS = RS

The solutions are -2 and -3.

53. $\dfrac{2}{s+3}-\dfrac{3}{s-2}=2$

$2(s-2)-3(s+3)=2(s+3)(s-2)$ ← Multiply both sides by $(s+3)(s-2)$.

$2s-4-3s-9=2\left(s^2+s-6\right)$

$-s-13=2s^2+2s-12$

$0=2s^2+3s+1$

$0=(2s+1)(s+1)$

$\therefore\ 2s+1=0$ or $s+1=0$

$s=-\dfrac{1}{2}$ $\qquad$ $s=-1$

Check

For $s=-\frac{1}{2}$,

LS	RS
$\dfrac{2}{s+3}-\dfrac{3}{s-2}$	2
$=\dfrac{2}{-\frac{1}{2}+3}-\dfrac{3}{-\frac{1}{2}-2}$	
$=\dfrac{2}{-\frac{1}{2}+\frac{6}{2}}-\dfrac{3}{-\frac{1}{2}-\frac{4}{2}}$	
$=\dfrac{2}{\frac{5}{2}}-\dfrac{3}{-\frac{5}{2}}$	
$=\dfrac{4}{5}+\dfrac{6}{5}$	
$=\dfrac{10}{5}$	
$=2$	

LS = RS

For $s=-1$,

LS	RS
$\dfrac{2}{s+3}-\dfrac{3}{s-2}$	2
$=\dfrac{2}{-1+3}-\dfrac{3}{-1-2}$	
$=\dfrac{2}{2}-\dfrac{3}{-3}$	
$=1+1$	
$=2$	

LS = RS

The roots are $-\frac{1}{2}$ and -1.

Section 2: Activity 2 (continued)

59. $x=\frac{1}{3}$ and $x=\frac{2}{3}$

$3x=1$ $\quad$ $3x=2$

$3x-1=0$ $\quad$ $3x-2=0$

$\therefore\ (3x-1)(3x-2)=0$

$9x^2-6x-3x+2=0$

$9x^2-9x+2=0$

A quadratic equation with roots $\frac{1}{3}$ and $\frac{2}{3}$ is $9x^2-9x+2=0$.

60. $x=-\frac{3}{2}$ and $x=\frac{1}{4}$

$2x=-3$ $\quad$ $4x=1$

$2x+3=0$ $\quad$ $4x-1=0$

$\therefore\ (2x+3)(4x-1)=0$

$8x^2-2x+12x-3=0$

$8x^2+10x-3=0$

A quadratic equation with roots $-\frac{3}{2}$ and $\frac{1}{4}$ is $8x^2+10x-3=0$.

62. $x=0$ and $x=\frac{4}{3}$

$3x=4$

$3x-4=0$

$\therefore\ x(3x-4)=0$

$3x^2-4x=0$

A quadratic equation with roots 0 and $\frac{4}{3}$ is $3x^2-4x=0$.

b. Textbook questions 69, 73, 75, 76, 81, 82, 86, 89, and 90 of "Applications and Problem Solving," pp. 161 and 162

69. a. When the object strikes the ground, $h(t)=0$.

$\therefore\ 0=-5t^2+9t+2$

b. $0 = -5t^2 + 9t + 2$

$5t^2 - 9t - 2 = 0$

$(5t+1)(t-2) = 0$

$\therefore\ 5t+1 = 0 \quad \text{or} \quad t-2 = 0$

$t = -\frac{1}{5} \qquad\qquad t = 2$

Because time is non-negative, $t = -\frac{1}{5}$ is an inadmissible value. Therefore, the time the object takes to strike the ground is 2 s.

73. Let x be the smaller number, and let $x+2$ be the larger number.

$\therefore\ x(x+2) = 288$

$x^2 + 2x - 288 = 0$

$(x+18)(x-16) = 0$

$\therefore\ x+18 = 0 \quad \text{or} \quad x-16 = 0$

$x = -18 \qquad\qquad x = 16$

If $x = -18$, then $x+2 = -16$.

If $x = 16$, then $x+2 = 18$.

Therefore, the two consecutive even numbers with a product of 288 are -18 and -16 or 16 and 18.

75. Area = length × width

$36 = x(x-5)$

$36 = x^2 - 5x$

$0 = x^2 - 5x - 36$

$0 = (x-9)(x+4)$

$\therefore\ x-9 = 0 \quad \text{or} \quad x+4 = 0$

$x = 9 \qquad\qquad x = -4$

Because length is non-negative, $x = -4$ is inadmissible. Therefore, the dimensions of the rectangle are 9 cm and $9-5 = 4$ cm.

Section 2: Activity 2 (continued)

76. Let x be the number. Therefore, its reciprocal is $\frac{1}{x}$.

$$\therefore\ x+\frac{1}{x}=\frac{13}{6}$$

$$x^2+1=\frac{13}{6}x \quad \longleftarrow \text{Multiply both sides by } x.$$

$$6x^2+6=13x \quad \longleftarrow \text{Multiply both sides by 6.}$$

$$6x^2-13x+6=0$$

$$(2x-3)(3x-2)=0$$

$$\therefore\ 2x-3=0 \quad \text{or} \quad 3x-2=0$$

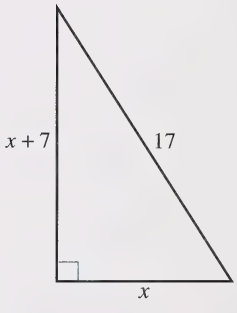
$$x=\frac{3}{2} \qquad\qquad x=\frac{2}{3}$$

If $x=\frac{3}{2}$, its reciprocal is $\frac{2}{3}$.

If $x=\frac{2}{3}$, its reciprocal is $\frac{3}{2}$.

Therefore, the number is either $\frac{3}{2}$ or $\frac{2}{3}$.

81. Let x be the shorter leg, and let $x+7$ be the longer leg.



Using the Pythogorean Theorem,

$$x^2+(x+7)^2=17^2$$

$$x^2+x^2+14x+49=289$$

$$2x^2+14x-240=0$$

$$2\left(x^2+7x-120\right)=0$$

$$2(x+15)(x-8)=0$$

$$\therefore\ x+15=0 \quad \text{or} \quad x-8=0$$

$$x=-15 \qquad\qquad x=8$$

Because length in non-negative, $x=-15$ is inadmissible. Therefore, the lengths of the legs are 8 cm and $8+7=15$ cm.

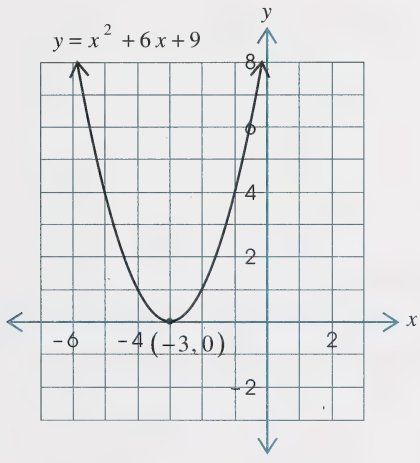
82. Number of diagonals = 44

$$\therefore \frac{n(n-3)}{2} = 44$$
$$n(n-3) = 88$$
$$n^2 - 3n - 88 = 0$$
$$(n-11)(n+8) = 0$$

$$\therefore n-11 = 0 \quad \text{or} \quad n+8 = 0$$
$$n = 11 \qquad\qquad n = -8$$

The number of diagonals must be positive. Therefore, there are 11 diagonals.

86. a.



There appears to be one root, $x = -3$.

b. $x^2 + 6x + 9 = 0$

$(x+3)(x+3) = 0$

$$\therefore x+3 = 0 \quad \text{or} \quad x+3 = 0$$
$$x = -3 \qquad\qquad x = -3$$

There are two equal real roots; they are both equal to -3.

Section 2: Activity 2 (continued)

89. a.

$$x = \frac{2}{3} \quad \text{and} \quad x = -\frac{1}{2}$$

$$3x = 2 \qquad 2x = -1$$

$$3x - 2 = 0 \qquad 2x + 1 = 0$$

$$\therefore\ (3x-2)(2x+1) = 0$$

$$6x^2 + 3x - 4x - 2 = 0$$

$$6x^2 - x - 2 = 0$$

A quadratic equation with roots $\frac{2}{3}$ and $-\frac{1}{2}$ is $6x^2 - x - 2 = 0$.

b. It is possible to write an infinite number of quadratic equations with the same roots. Simply multiply the equation in the answer to question 89.a. by any non-zero constant.

For example,

$$3\left(6x^2 - x - 2\right) = 0$$

$$18x^2 - 3x - 6 = 0$$

This has the same pair of roots.

90. a. Because $x = -3$ is a root, substitute it into the original equation and solve for m.

$$3x^2 + mx + 3 = 0$$

$$3(-3)^2 + m(-3) + 3 = 0$$

$$27 - 3m + 3 = 0$$

$$-3m = -30$$

$$m = 10$$

The value of m is 10.

b. If $m = 10$, then

$$3x^2 + mx + 3 = 0$$
$$3x^2 + 10x + 3 = 0$$
$$(x+3)(3x+1) = 0 \quad \text{◄— Factor to find both roots.}$$

$$\therefore\ x+3 = 0 \quad \text{or} \quad 3x+1 = 0$$
$$x = -3 \qquad\qquad x = -\frac{1}{3}$$

One root is -3; the other root is $-\frac{1}{3}$.

5. Textbook questions 1 and 2 of "PATTERN POWER," p. 162

1. The sum of the numbers in the first row is 53; the sum of the numbers in the second row is 54; the sum of the numbers in the third row is 55; and so on.

2. According to the pattern, the sum of the numbers in the fifth row should be 57.

$$\therefore\ 15+13+4+\square = 57$$
$$32+\square = 57$$
$$\square = 25$$

Therefore, the missing number is 25.

Section 2: Activity 3

1.
$$(5-h)^2 = 16$$
$$25-10h+h^2 = 16$$
$$h^2-10h+9 = 0$$
$$(h-1)(h-9) = 0$$

$$\therefore\ h-1 = 0 \quad \text{or} \quad h-9 = 0$$
$$h = 1 \qquad\qquad h = 9$$

Yes, the solution is the same.

Section 2: Activity 3 (continued)

2. a. Textbook questions a. and b. of "Explore: Use the Diagram," p. 163

a. The side length of the whole site is $4x$.

b. The area of the site is $16x^2$.

b. Textbook questions 1 to 7 of "Inquire," pp. 163 and 164

1. $16x^2 = 144$

2. $16x^2 = 144$

$x^2 = 9$

3. $x^2 = 9$

$x = \pm\sqrt{9}$

$= \pm 3$

4. Because length is non-negative, $x = -3$ must be rejected.

5. $(10x)^2 = 400$

$100x^2 = 400$

$x^2 = 4$

$x = \pm\sqrt{4}$

$= \pm 2$

Again, $x = -2$ must be rejected. Therefore, the sides of each small grid square are 2 m.

6. a. Each small square is s^2 in area. Therefore, $64s^2$ represents the area of the chessboard.

b. Area $= 576$

$64s^2 = 576$

$s^2 = 9$

$s = \pm\sqrt{9}$

$= \pm 3$

$s = -3$ is inadmissible and must be rejected.

Each side of the small square is 3 cm.

c. Area $= 784$

$64s^2 = 784$

$s^2 = 16$

$s = \pm\sqrt{16}$

$= \pm 4$

$s = -4$ is inadmissible.

Each side of the small square is 4 cm.

7. a. $x^2 = 36$

$x = \pm\sqrt{36}$

$= \pm 6$

b. $y^2 = 49$

$y = \pm\sqrt{49}$

$= \pm 7$

c. $w^2 = 121$

$w = \pm\sqrt{121}$

$= \pm 11$

d. $2n^2 = 8$

$n^2 = 4$

$n = \pm\sqrt{4}$

$= \pm 2$

e. $4t^2 = 100$

$t^2 = 25$

$t = \pm\sqrt{25}$

$= \pm 5$

f. $50r^2 = 72$

$r^2 = \frac{72}{50}$

$r^2 = \frac{36}{25}$

$r = \pm\sqrt{\frac{36}{25}}$

$= \pm\frac{6}{5}$

3. a. Textbook questions 4, 11, 18, 23, 40, 43, 51, 57, 65, and 69 of "Practice," pp. 165 and 166

4. $\frac{a^2}{2} = 8$

$a^2 = 16$

$a = \pm\sqrt{16}$

$= \pm 4$

The roots are 4 and -4.

Section 2: Activity 3 (continued)

11. $\frac{z^2-1}{5}=7$

$z^2-1=35$

$z^2=36$

$z=\pm\sqrt{36}$

$=\pm 6$

Check

For $z=6$,

LS	RS
$\frac{z^2-1}{5}$	7
$=\frac{6^2-1}{5}$	
$=\frac{36-1}{5}$	
$=\frac{35}{5}$	
$=7$	

LS = RS

For $z=-6$,

LS	RS
$\frac{z^2-1}{5}$	7
$=\frac{(-6)^2-1}{5}$	
$=\frac{36-1}{5}$	
$=\frac{35}{5}$	
$=7$	

LS = RS

The roots are 6 and -6.

18. $10c^2+2.2=7.1$

$10c^2=4.9$

$c^2=0.49$

$c=\pm\sqrt{0.49}$

$=\pm 0.7$

Check

For $c = 0.7$,

LS	RS
$10c^2 + 2.2$	7.1
$= 10(0.7)^2 + 2.2$	
$= 10(0.49) + 2.2$	
$= 4.9 + 2.2$	
$= 7.1$	
LS = RS	

For $c = -0.7$,

LS	RS
$10c^2 + 2.2$	7.1
$= 10(-0.7)^2 + 2.2$	
$= 10(0.49) + 2.2$	
$= 4.9 + 2.2$	
$= 7.1$	
LS = RS	

The roots are 0.7 and -0.7.

23. $12 + x^2 = 52$

$$x^2 = 40$$
$$x = \pm\sqrt{40}$$
$$\doteq \pm 6.32$$

The solutions are approximately 6.32 and -6.32.

40. $10x^2 - 11 = 5$

$$10x^2 = 16$$
$$x^2 = \frac{16}{10}$$
$$x = \pm\sqrt{\frac{16}{10}}$$
$$= \pm\frac{4}{\sqrt{10}}$$
$$= \pm\frac{4}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}}$$
$$= \pm\frac{4\sqrt{10}}{10}$$
$$= \pm\frac{2\sqrt{10}}{5}$$

The solutions are $\frac{2\sqrt{10}}{5}$ and $-\frac{2\sqrt{10}}{5}$.

Section 2: Activity 3 (continued)

43. $\dfrac{2x^2}{3} = 3$

$2x^2 = 9$

$x^2 = \dfrac{9}{2}$

$x = \pm\sqrt{\dfrac{9}{2}}$

$= \pm\dfrac{3}{\sqrt{2}}$

$= \pm\dfrac{3}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \pm\dfrac{3\sqrt{2}}{2}$

The solutions are $\frac{3\sqrt{2}}{2}$ and $-\frac{3\sqrt{2}}{2}$.

51. $(2x-3)^2 = 25$

$2x-3 = \pm\sqrt{25}$

$2x-3 = \pm 5$

$\therefore\ 2x-3 = 5$ or $2x-3 = -5$

$2x = 8$ $\qquad$ $2x = -2$

$x = 4$ $\qquad$ $x = -1$

Check

For $x = 4$,

LS	RS
$(2x-3)^2$	25
$=[2(4)-3]^2$	
$=(8-3)^2$	
$=(5)^2$	
$=25$	

LS = RS

For $x = -1$,

LS	RS
$(2x-3)^2$	25
$=[2(-1)-3]^2$	
$=(-2-3)^2$	
$=(-5)^2$	
$=25$	

LS = RS

The roots are 4 and -1.

57. $(3y+2)^2 = 0$

$3y+2=0$

$3y=-2$

$y=-\frac{2}{3}$

The root is $-\frac{2}{3}$.

65. $(x-5)^2 = 3$

$x-5=\pm\sqrt{3}$

$\therefore\ x-5=\sqrt{3}$ or $x-5=-\sqrt{3}$

$x=5+\sqrt{3}$ $\qquad$ $x=5-\sqrt{3}$

$\doteq 6.73$ $\qquad$ $\doteq 3.27$

The solutions are $5+\sqrt{3}$ and $5-\sqrt{3}$, or approximately 6.73 and 3.27.

69. $4(y+3)^2+3=10$

$4(y+3)^2=7$

$(y+3)^2=\frac{7}{4}$

$y+3=\pm\sqrt{\frac{7}{4}}$

$y+3=\pm\frac{\sqrt{7}}{2}$

$\therefore\ y+3=\frac{\sqrt{7}}{2}$ or $y+3=-\frac{\sqrt{7}}{2}$

$y=-3+\frac{\sqrt{7}}{2}$ $\qquad$ $y=-3-\frac{\sqrt{7}}{2}$

$=\frac{-6+\sqrt{7}}{2}$ $\qquad$ $=\frac{-6-\sqrt{7}}{2}$

$\doteq -1.68$ $\qquad$ $\doteq -4.32$

The solutions are $\frac{-6+\sqrt{7}}{2}$ and $\frac{-6-\sqrt{7}}{2}$, or approximately -1.68 and -4.32.

Section 2: Activity 3 (continued)

b. Textbook questions 74, 75, 76, 81, 85, 89.e., and 91 of "Applications and Problem Solving," pp. 166 and 167

74. a. The length is $2w$.

b. $\text{Area} = \text{length} \times \text{width}$

$= 2w(w)$

$= 2w^2$

c. $2w^2 = 800$

$w^2 = 400$

$w = \pm\sqrt{400}$

$w = \pm 20$

Because width is positive, $w = 20$ and $\ell = 2(20) = 40$.

Therefore, the dimensions of the flag are 20 cm × 40 cm.

75. a. When the object reaches the ground, $h = 0$.

$\therefore\ h = -4.9t^2 + 196$

$0 = -4.9t^2 + 196$

$4.9t^2 = 196$

$t^2 = 40$

$t = \pm\sqrt{40}$

$= \pm 2\sqrt{10}$

Because time is positive, the object reaches the ground in $2\sqrt{10}$ s.

b. The object reaches the ground in approximately 6.3 s.

76. a. $T^2 = 4\ell$

$T^2 = 4(1)$

$T^2 = 4$

$T = \pm\sqrt{4}$

$= \pm 2$

Because the pendulum's period is non-negative, the period is 2 s.

b. $T^2 = 4\ell$

$T^2 = 4(27.4)$

$T^2 = 109.6$

$T = \pm\sqrt{109.6}$

$\doteq \pm 10.5$

The period of the pendulum in Portland, Oregon, is approximately 10.5 s.

81. Let the number be x; therefore, its reciprocal is $\frac{1}{x}$.

$\therefore x + \frac{1}{x} = 5x$

$x^2 + 1 = 5x^2$ ← Multiply both sides by x.

$1 = 4x^2$

$4x^2 = 1$

$x^2 = \frac{1}{4}$

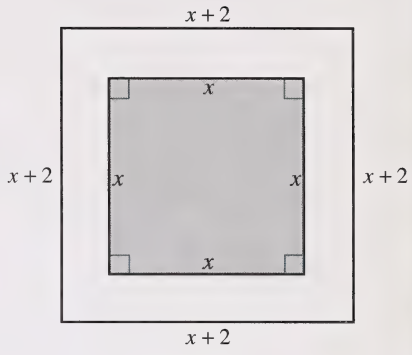
$x = \pm\sqrt{\frac{1}{4}}$

$= \pm\frac{1}{2}$

The number is either $\frac{1}{2}$ or $-\frac{1}{2}$.

Section 2: Activity 3 (continued)

85. Let x be the length of a side of the photograph, and let $x+2$ be the length of a side of the framed photograph.



$$\therefore\ (x+2)^2 = 169$$
$$x+2 = \pm 13$$

$$\therefore\ x+2 = 13 \quad \text{or} \quad x+2 = -13$$
$$x = 11 \qquad\qquad x = -15$$

Because length is non-negative, the dimensions of the photograph are 11 cm × 11 cm.

89. e. $\dfrac{1}{(a-2)^2} = 4$

$$1 = 4(a-2)^2$$
$$\frac{1}{4} = (a-2)^2$$
$$(a-2)^2 = \frac{1}{4}$$
$$a-2 = \pm\frac{1}{2}$$

$$\therefore\ a-2 = \frac{1}{2} \quad \text{or} \quad a-2 = -\frac{1}{2}$$
$$a = 2\frac{1}{2} \qquad\qquad a = 1\frac{1}{2}$$

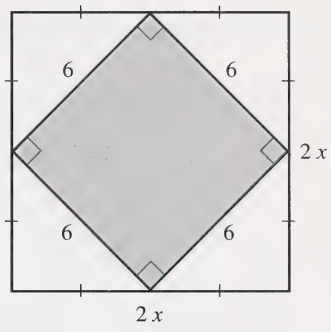
The solutions are $2\frac{1}{2}$ and $1\frac{1}{2}$.

91. a. $A = s^2$

$36 = s^2$

$s^2 = 36$

$s = 6$



The shaded square is 6 cm on a side.

Let $2x$ be the length of the sides of the large square (as shown in the diagram on the right).

Using the Pythogorean Theorem,

$$x^2 + x^2 = 6^2$$
$$2x^2 = 36$$
$$x^2 = 18$$
$$x = \pm\sqrt{18}$$
$$= \pm 3\sqrt{2}$$

Because length is non-negative, the large square is $2\left(3\sqrt{2}\right) = 6\sqrt{2}$ cm, or approximately 8.5 cm, on a side.

b. Area $= \left(6\sqrt{2}\right)^2$

$= 36(2)$

$= 72$

The large square is 72 cm^2 in area.

4. Textbook question "WORD POWER," p. 167

Answers may vary. Two sample answers are given.

SILK	SILK
MILK	SILL
MILE	FILL
MOLE	FILM
MORE	FIRM
WORE	FORM
WORM	WORM

Section 2: Activity 4

1. a. Textbook questions a. and b. of "Explore: Use a Diagram," p. 168

a. The length of the forum is $x+120$.

$\therefore$ Area = length $\times$ width

$= (x+120)x$

$= x^2+120x$

b. $x^2+120x=6400$

b. Textbook questions 1 to 7 of "Inquire," pp. 168 and 169

1. a. $x^2+120x=6400$

$x^2+120x+3600=6400+3600$

$x^2+120x+3600=10\,000$

Divide the coefficient of x by 2, and square the result. Therefore, add $(120\div 2)^2$ or 3600 to both sides.

b. The number, 3600, must be added to both sides in order to balance the equation.

2. $(x+60)^2=10\,000$

3. $x+60=\pm\sqrt{10\,000}$

$x+60=\pm 100$

4. $x+60=100$ or $x+60=-100$

$x=40$ $\qquad$ $x=-140$

5. The negative value, $x=-140$, should be discarded because width is positive.

6. a. The width is 40 m.

b. The length is $40+120=160$ m.

7. a. $x^2+4x=12$

$x^2+4x+4=12+4$ $\left(\frac{4}{2}\right)^2=4$

$(x+2)^2=16$

$x+2=\pm\sqrt{16}$

$x+2=\pm4$

$\therefore\ x+2=4$ or $x+2=-4$

$x=2$ $x=-6$

The roots are 2 and -6.

b. $x^2-2x=3$

$x^2-2x+1=3+1$ $\left(\frac{-2}{2}\right)^2=1$

$(x-1)^2=4$

$x-1=\pm2$

$\therefore\ x-1=2$ or $x-1=-2$

$x=3$ $x=-1$

The roots are 3 and -1.

c. $x^2+6x=-8$

$x^2+6x+9=-8+9$ $\left(\frac{6}{2}\right)^2=9$

$(x+3)^2=1$

$x+3=\pm1$

$\therefore\ x+3=1$ or $x+3=-1$

$x=-2$ $x=-4$

The roots are -2 and -4.

Section 2: Activity 4 (continued)

2. a. Textbook questions 4, 11, 15, 20, 24, 27, 33, 37, 41, 43, 49, and 52 of "Practice," p. 172

4. $k = \left(\frac{-14}{2}\right)^2$

$= (-7)^2$

$= 49$

$\therefore\ w^2 - 14w + 49 = (w-7)^2$

11. $k = \left(-\frac{1}{2} \div 2\right)^2$

$= \left(-\frac{1}{2} \times \frac{1}{2}\right)^2$

$= \left(-\frac{1}{4}\right)^2$

$= \frac{1}{16}$

$\therefore\ x^2 - \frac{1}{2}x + \frac{1}{16} = \left(x - \frac{1}{4}\right)^2$

15. $(x+3)^2 = 9$

$x + 3 = \pm 3$

$\therefore\ x + 3 = 3$ or $x + 3 = -3$

$x = 0$ $\qquad$ $x = -6$

The roots are 0 and -6.

20. $(y-4)^2-12=0$

$(y-4)^2=12$

$y-4=\pm\sqrt{12}$

$y-4=\pm 2\sqrt{3}$

$\therefore\ y-4=2\sqrt{3}$ or $y-4=-2\sqrt{3}$

$y=4+2\sqrt{3}$ $\qquad$ $y=4-2\sqrt{3}$

The roots are $4+2\sqrt{3}$ and $4-2\sqrt{3}$.

24. $\left(c+\frac{3}{2}\right)^2=\frac{3}{8}$

$c+\frac{3}{2}=\pm\sqrt{\frac{3}{8}}$

$c+\frac{3}{2}=\pm\frac{\sqrt{3}}{2\sqrt{2}}$

$c+\frac{3}{2}=\pm\frac{\sqrt{3}}{2\sqrt{2}}\bullet\frac{\sqrt{2}}{\sqrt{2}}$ ← Rationalize.

$c+\frac{3}{2}=\pm\frac{\sqrt{6}}{4}$

$\therefore\ c+\frac{3}{2}=\frac{\sqrt{6}}{4}$ or $c+\frac{3}{2}=-\frac{\sqrt{6}}{4}$

$c=-\frac{3}{2}+\frac{\sqrt{6}}{4}$ $\qquad$ $c=-\frac{3}{2}-\frac{\sqrt{6}}{4}$

The roots are $-\frac{3}{2}+\frac{\sqrt{6}}{4}$ and $-\frac{3}{2}-\frac{\sqrt{6}}{4}$.

27. $x^2+6x+4=0$

$x^2+6x=-4$

$x^2+6x+9=-4+9$

$(x+3)^2=5$ $\qquad$ $\therefore\ x+3=\sqrt{5}$ or $x+3=-\sqrt{5}$

$x+3=\pm\sqrt{5}$ $\qquad$ $x=-3+\sqrt{5}$ $\qquad$ $x=-3-\sqrt{5}$

The roots are $-3+\sqrt{5}$ and $-3-\sqrt{5}$.

Section 2: Activity 4 (continued)

33. $x-3=-x^2$

$x^2+x-3=0$

$x^2+x=3$

$x^2+x+\frac{1}{4}=3+\frac{1}{4}$ $\qquad \left(\frac{1}{2}\right)^2=\frac{1}{4}$

$\left(x+\frac{1}{2}\right)^2=\frac{13}{4}$

$x+\frac{1}{2}=\pm\frac{\sqrt{13}}{2}$

$\therefore\ x+\frac{1}{2}=\frac{\sqrt{13}}{2}$ or $x+\frac{1}{2}=-\frac{\sqrt{13}}{2}$

$x=-\frac{1}{2}+\frac{\sqrt{13}}{2}$ $\qquad x=-\frac{1}{2}-\frac{\sqrt{13}}{2}$

The roots are $-\frac{1}{2}+\frac{\sqrt{13}}{2}$ and $-\frac{1}{2}-\frac{\sqrt{13}}{2}$.

37. $4t^2+10t+5=0$

$4t^2+10t=-5$

$t^2+\frac{5}{2}t=-\frac{5}{4}$

$t^2+\frac{5}{2}t+\frac{25}{16}=-\frac{5}{4}+\frac{25}{16}$ $\qquad \left(\frac{5}{2}\times\frac{1}{2}\right)^2=\frac{25}{16}$

$\left(t+\frac{5}{4}\right)^2=\frac{5}{16}$

$t+\frac{5}{4}=\pm\frac{\sqrt{5}}{4}$

$\therefore\ t+\frac{5}{4}=\frac{\sqrt{5}}{4}$ or $t+\frac{5}{4}=-\frac{\sqrt{5}}{4}$

$t=-\frac{5}{4}+\frac{\sqrt{5}}{4}$ $\qquad t=-\frac{5}{4}-\frac{\sqrt{5}}{4}$

The roots are $-\frac{5}{4}+\frac{\sqrt{5}}{4}$ and $-\frac{5}{4}-\frac{\sqrt{5}}{4}$.

41. $2x-6=-5x^2$

$5x^2+2x-6=0$

$5x^2+2x=6$

$x^2+\frac{2}{5}x=\frac{6}{5}$

$x^2+\frac{2}{5}x+\frac{1}{25}=\frac{6}{5}+\frac{1}{25}$

$\left(\frac{2}{5}\times\frac{1}{2}\right)^2=\frac{1}{25}$

$\left(x+\frac{1}{5}\right)^2=\frac{31}{25}$

$x+\frac{1}{5}=\pm\frac{\sqrt{31}}{5}$

$\therefore\ x+\frac{1}{5}=+\frac{\sqrt{31}}{5}$ or $x+\frac{1}{5}=-\frac{\sqrt{31}}{5}$

$x=-\frac{1}{5}+\frac{\sqrt{31}}{5}$ $\qquad$ $x=-\frac{1}{5}-\frac{\sqrt{31}}{5}$

The roots are $-\frac{1}{5}+\frac{\sqrt{31}}{5}$ and $-\frac{1}{5}-\frac{\sqrt{31}}{5}$.

43. $\frac{1}{2}x^2+x-13=0$

$x^2+2x-26=0$

$x^2+2x=26$

$x^2+2x+1=26+1$

$(x+1)^2=27$

$x+1=\pm\sqrt{27}$

$x+1=\pm3\sqrt{3}$

$\therefore\ x+1=3\sqrt{3}$ or $x+1=-3\sqrt{3}$

$x=-1+3\sqrt{3}$ $\qquad$ $x=-1-3\sqrt{3}$

The roots are $-1+3\sqrt{3}$ and $-1-3\sqrt{3}$.

Section 2: Activity 4 (continued)

49.

$$0 = 2r^2 - 8r + 3$$
$$-3 = 2r^2 - 8r$$
$$2r^2 - 8r = -3$$
$$r^2 - 4r = -\frac{3}{2}$$
$$r^2 - 4r + 4 = -\frac{3}{2} + 4$$
$$(r-2)^2 = \frac{5}{2}$$
$$r - 2 = \pm\sqrt{\frac{5}{2}}$$
$$r - 2 = \pm\frac{\sqrt{5}}{\sqrt{2}} \bullet \frac{\sqrt{2}}{\sqrt{2}}$$
$$r - 2 = \pm\frac{\sqrt{10}}{2}$$

$\therefore\ r - 2 = \dfrac{\sqrt{10}}{2}$ or $r - 2 = -\dfrac{\sqrt{10}}{2}$

$r = 2 + \dfrac{\sqrt{10}}{2}$ $\qquad$ $r = 2 - \dfrac{\sqrt{10}}{2}$

$\doteq 3.58$ $\qquad$ $\doteq 0.42$

The roots are approximately 3.58 and 0.42.

52. $\dfrac{2}{3}x^2 - 2x - 3 = 0$

$$2x^2 - 6x - 9 = 0$$
$$2x^2 - 6x = 9$$
$$x^2 - 3x = \frac{9}{2}$$
$$x^2 - 3x + \frac{9}{4} = \frac{9}{2} + \frac{9}{4}$$
$$\left(x - \frac{3}{2}\right)^2 = \frac{27}{4}$$
$$x - \frac{3}{2} = \pm\sqrt{\frac{27}{4}}$$
$$x - \frac{3}{2} = \pm\frac{3\sqrt{3}}{2}$$

$\left(\dfrac{-3}{2}\right)^2 = \dfrac{9}{4}$

$$\therefore\ x-\frac{3}{2}=\frac{3\sqrt{3}}{2} \qquad \text{or} \qquad x-\frac{3}{2}=-\frac{3\sqrt{3}}{2}$$

$$x=\frac{3}{2}+\frac{3\sqrt{3}}{2} \qquad x=\frac{3}{2}-\frac{3\sqrt{3}}{2}$$

$$x\doteq 4.10 \qquad x\doteq -1.10$$

The roots are approximately 4.10 and -1.10.

b. Textbook questions 57, 58, 60, 63, 67.a., 68, 72.a., and 72.d. of "Applications and Problem Solving," pp. 172 and 173

57. Let x be the width, and let $x+2$ be the length.

$$\therefore \text{ Area} = \text{length} \times \text{width}$$

$$20=(x+2)x$$

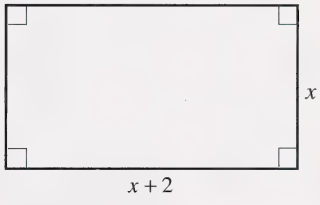
$$x(x+2)=20$$

$$x^2+2x=20$$

$$x^2+2x+1=20+1 \qquad \longleftarrow \text{Complete the square.}$$

$$(x+1)^2=21$$

$$x+1=\pm\sqrt{21}$$



$$\therefore\ x+1=\sqrt{21} \qquad \text{or} \qquad x+1=-\sqrt{21}$$

$$x=-1+\sqrt{21} \qquad x=-1-\sqrt{21}$$

$$\doteq 3.6 \qquad =-5.6$$

Because width is non-negative, the width of the rectangle is approximately 3.6 m.

$$\therefore \text{ length} = x+2$$

$$\doteq 3.6+2$$

$$\doteq 5.6$$

Therefore, the dimensions of the rectangle are approximately 3.6 m × 5.6 m.

Section 2: Activity 4 (continued)

58. Let x be the increase to each dimension; therefore, the width is $4+x$ and the length is $6+x$.

$$\begin{aligned}
\therefore \text{Area} &= \text{length} \times \text{width} \\
50 &= (6+x)(4+x) \\
(6+x)(4+x) &= 50 \\
24+6x+4x+x^2 &= 50 \\
x^2+10x+24 &= 50 \\
x^2+10x &= 26 \\
x^2+10x+25 &= 26+25 \qquad \longleftarrow \text{Complete the square.} \\
(x+5)^2 &= 51 \\
x+5 &= \pm\sqrt{51}
\end{aligned}$$

$$\therefore x+5=\sqrt{51} \quad \text{or} \quad x+5=-\sqrt{51}$$

$$x=-5+\sqrt{51} \qquad x=-5-\sqrt{51}$$

$$\doteq 2.1 \qquad \doteq -12.1$$

Because the length increases, $x>0$; therefore, the increase in each dimension is approximately 2.1 m.

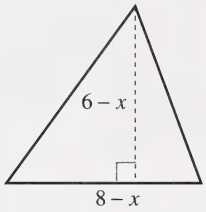
$$\therefore \text{width} = 4+x \qquad \therefore \text{length} = 6+x$$

$$\doteq 4+2.1 \qquad \doteq 6+2.1$$

$$\doteq 6.1 \qquad \doteq 8.1$$

Therefore, the rectangle's dimensions are approximately 6.1 m × 8.1 m.

60. Let x be the decrease (in centimetres) in the height and the base; therefore, the base is $8-x$ and the height is $6-x$.



$$\therefore \text{Area} = \frac{1}{2}\,\text{base} \times \text{height}$$

$$20 = \frac{1}{2}(8-x)(6-x)$$

$$40 = (8-x)(6-x)$$

$$(8-x)(6-x) = 40$$

$$48 - 8x - 6x + x^2 = 40$$

$$x^2 - 14x + 48 = 40$$

$$x^2 - 14x = -8$$

$$x^2 - 14x + 49 = -8 + 49 \quad \longleftarrow \text{Complete the square.}$$

$$(x-7)^2 = 41$$

$$x - 7 = \pm\sqrt{41}$$

$$\therefore x - 7 = \sqrt{41} \quad \text{or} \quad x - 7 = -\sqrt{41}$$

$$x = 7 + \sqrt{41} \qquad x = 7 - \sqrt{41}$$

$$\doteq 13.4 \qquad \doteq 0.6$$

Because the original height was 6 cm, $x < 6$ cm. Therefore, the decrease in each dimension is approximately 0.6 cm. The base is about 7.4 cm and the height is about 5.4 cm.

Section 2: Activity 4 (continued)

63. a. Let x be one number, and let y be the other number.

$$\therefore\ x+y=14 \quad (1)$$
$$xy=37 \quad (2)$$

Solve (1) for y.

$$x+y=14$$
$$y=14-x$$

Substitute $y=14-x$ into (2).

$$x(14-x)=37$$
$$14x-x^2=37$$
$$0=x^2-14x+37$$
$$x^2-14x+37=0$$
$$x^2-14x=-37$$
$$x^2-14x+49=-37+49$$
$$(x-7)^2=12$$
$$x-7=\pm\sqrt{12}$$
$$x-7=\pm 2\sqrt{3}$$

$$\therefore\ x-7=2\sqrt{3} \quad \text{or} \quad x-7=-2\sqrt{3}$$
$$x=7+2\sqrt{3} \qquad\qquad x=7-2\sqrt{3}$$

If $x=7+2\sqrt{3}$, then

$$y=14-\left(7+2\sqrt{3}\right)$$
$$=14-7-2\sqrt{3}$$
$$=7-2\sqrt{3}$$

If $x=7-2\sqrt{3}$, then

$$y=14-\left(7-2\sqrt{3}\right)$$
$$=14-7+2\sqrt{3}$$
$$=7+2\sqrt{3}$$

The numbers are $7-2\sqrt{3}$ or $7+2\sqrt{3}$.

b. The numbers are approximately 3.536 and 10.464.

67. a.
$$\frac{3}{x}-\frac{4}{2x}=x+1$$
$$\frac{3}{x}-\frac{2}{x}=x+1$$
$$\frac{1}{x}=x+1 \quad \longleftarrow \text{Multiply both sides by } x.$$
$$1=x^2+x$$
$$x^2+x=1$$
$$x^2+x+\frac{1}{4}=1+\frac{1}{4}$$
$$\left(x+\frac{1}{2}\right)^2=\frac{5}{4}$$
$$x+\frac{1}{2}=\pm\frac{\sqrt{5}}{2}$$

$$\therefore\ x+\frac{1}{2}=\frac{\sqrt{5}}{2} \quad \text{or} \quad x+\frac{1}{2}=-\frac{\sqrt{5}}{2}$$
$$x=-\frac{1}{2}+\frac{\sqrt{5}}{2} \qquad x=-\frac{1}{2}-\frac{\sqrt{5}}{2}$$

Restrictions: $x\neq 0$

68. a. When the ball strikes the ground, $h(t)=0$.

$$\therefore\ h(t)=-5t^2+20t+2$$
$$0=-5t^2+20t+2$$
$$5t^2-20t=2$$
$$t^2-4t=\frac{2}{5}$$
$$t^2-4t+4=\frac{2}{5}+4$$
$$(t-2)^2=\frac{22}{5}$$
$$t-2=\pm\sqrt{\frac{22}{5}} \quad \longleftarrow \text{Rationalize.}$$
$$t-2=\pm\frac{\sqrt{110}}{5}$$

Section 2: Activity 4 (continued)

$$\therefore\ t-2=\frac{\sqrt{110}}{5} \quad \text{or} \quad t-2=-\frac{\sqrt{110}}{5}$$

$$t=2+\frac{\sqrt{110}}{5} \qquad t=2-\frac{\sqrt{110}}{5}$$

$$\doteq 4.1 \qquad =-0.1$$

Because time is non-negative, the ball strikes the ground after approximately 4.1 s.

b. Determine when $h(t)=17$ m.

$$17=-5t^2+20t+2$$
$$5t^2-20t=-15$$
$$t^2-4t=-3$$
$$t^2-4t+4=-3+4$$
$$(t-2)^2=1$$
$$t-2=\pm 1$$

$$\therefore\ t-2=1 \quad \text{or} \quad t-2=-1$$
$$t=3 \qquad t=1$$

The football is at least 17 m above the ground when $1\text{ s}\le t\le 3\text{ s}$, a total of 2 s.

72. a. $x=\sqrt{5}$ and $x=-\sqrt{5}$

$x-\sqrt{5}=0 \qquad x+\sqrt{5}=0$

$$\therefore\ \left(x-\sqrt{5}\right)\left(x+\sqrt{5}\right)=0$$
$$x^2-5=0$$

A quadratic equation with roots $\sqrt{5}$ and $-\sqrt{5}$ is $x^2-5=0$.

d. $x=3+\frac{\sqrt{13}}{2}$ and $x=3-\frac{\sqrt{13}}{2}$

$x-3=\frac{\sqrt{13}}{2} \qquad x-3=-\frac{\sqrt{13}}{2}$

$2x-6=\sqrt{13} \qquad 2x-6=-\sqrt{13}$

$2x-6-\sqrt{13}=0 \qquad 2x-6+\sqrt{13}=0$

$$\therefore \left(2x-6-\sqrt{13}\right)\left(2x-6+\sqrt{13}\right)=0$$

$$2x\left(2x-6+\sqrt{13}\right)-6\left(2x-6+\sqrt{13}\right)-\sqrt{13}\left(2x-6+\sqrt{13}\right)=0$$

$$4x^2-12x+2\sqrt{13}x-12x+36-6\sqrt{13}-2\sqrt{13}x+6\sqrt{13}-13=0$$

$$4x^2-24x+23=0$$

An equation with roots $3+\frac{\sqrt{13}}{2}$ and $3-\frac{\sqrt{13}}{2}$ is $4x^2-24x+23=0$.

3. Textbook question "PATTERN POWER," p. 173

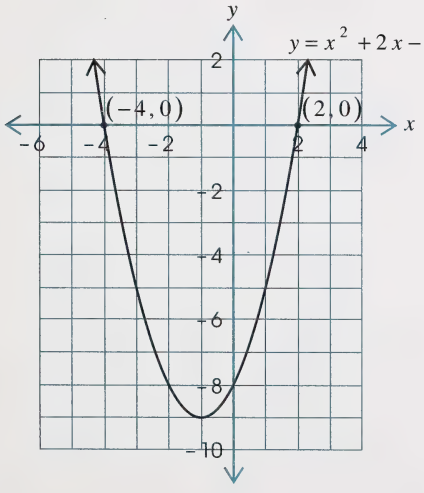
3 2 / 9 109 2 / 9 3 / 2 1	9322 − 9213 = 109	
7 1 / 4 898 0 / 3 2 / 8 1	4710 − 3812 = 898	
0 3 / 6 415 3 / 5 **8** / 6 1	6033 − 5618 = 415	

Section 2: Follow-up Activities

Extra Help

Textbook questions 1, 5, 9, 12, 16, 18, and 20 of "Chapter Check," p. 234

1.



The roots are -4 and 2, the x-intercepts of the graph.

Extra Help (continued)

5. $q^2+2q-15=0$

$(q+5)(q-3)=0$

$\therefore\ q+5=0 \quad \text{or} \quad q-3=0$

$q=-5 \qquad\qquad q=3$

Check

For $q=-5$,

LS	RS
$q^2+2q-15$	0
$=(-5)^2+2(-5)-15$	
$=25-10-15$	
$=0$	

LS = RS

For $q=3$,

LS	RS
$q^2+2q-15$	0
$=3^2+2(3)-15$	
$=9+6-15$	
$=0$	

LS = RS

The roots are -5 and 3.

9. $x-2=\frac{6}{x}-3$

$x^2-2x=6-3x$ ← Multiply both sides by x.

$x^2+x-6=0$ ← Write the equation in standard form.

$(x+3)(x-2)=0$

$\therefore\ x+3=0$ or $x-2=0$

$x=-3$ $x=2$

Check

For $x=-3$,

LS	RS
$x-2$	$\frac{6}{x}-3$
$=-3-2$	$=\frac{6}{-3}-3$
$=-5$	$=-2-3$
	$=-5$

LS = RS

For $x=2$,

LS	RS
$x-2$	$\frac{6}{x}-3$
$=2-2$	$=\frac{6}{2}-3$
$=0$	$=3-3$
	$=0$

LS = RS

The roots are -3 and 2.

12. $x=-\frac{1}{2}$ and $x=\frac{3}{4}$

$2x=-1$ $4x=3$

$2x+1=0$ $4x-3=0$

$\therefore\ (2x+1)(4x-3)=0$

$8x^2-6x+4x-3=0$

$8x^2-2x-3=0$

An equation with $-\frac{1}{2}$ and $\frac{3}{4}$ as roots is $8x^2-2x-3=0$.

Extra Help (continued)

16. $3(y-4)^2 = 12$

$(y-4)^2 = 4$

$y-4 = \pm 2$

$\therefore\ y-4 = 2$ or $y-4 = -2$

$y = 6$ $\qquad$ $y = 2$

Check

For $y = 6$,

LS	RS
$3(y-4)^2$	12
$=3(6-4)^2$	
$=3(2)^2$	
$=3(4)$	
$=12$	

LS = RS

For $y = 2$,

LS	RS
$3(y-4)^2$	12
$=3(2-4)^2$	
$=3(-2)^2$	
$=3(4)$	
$=12$	

LS = RS

The roots are 2 and 6.

18. $y^2 - 3y - 18 = 0$

$y^2 - 3y = 18$

$y^2 - 3y + \frac{9}{4} = 18 + \frac{9}{4}$ $\qquad$ $\left(\frac{-3}{2}\right)^2 = \frac{9}{4}$

$\left(y - \frac{3}{2}\right)^2 = \frac{81}{4}$

$y - \frac{3}{2} = \pm\frac{9}{2}$

$\therefore\ y - \frac{3}{2} = \frac{9}{2}$ or $y - \frac{3}{2} = -\frac{9}{2}$

$y = \frac{12}{2}$ $\qquad$ $y = -\frac{6}{2}$

$= 6$ $\qquad$ $= -3$

The roots are 6 and -3.

20.
$$2x = 5x^2 - 1$$
$$0 = 5x^2 - 2x - 1$$
$$5x^2 - 2x = 1$$
$$x^2 - \frac{2}{5}x = \frac{1}{5}$$
$$x^2 - \frac{2}{5}x + \frac{1}{25} = \frac{1}{5} + \frac{1}{25}$$
$$\left(x - \frac{1}{5}\right)^2 = \frac{6}{25}$$
$$x - \frac{1}{5} = \pm\frac{\sqrt{6}}{5}$$

$$\left(-\frac{2}{5} \div 2\right)^2 = \left(-\frac{\cancel{2}}{5} \times \frac{1}{\cancel{2}}\right)^2 = \frac{1}{25}$$

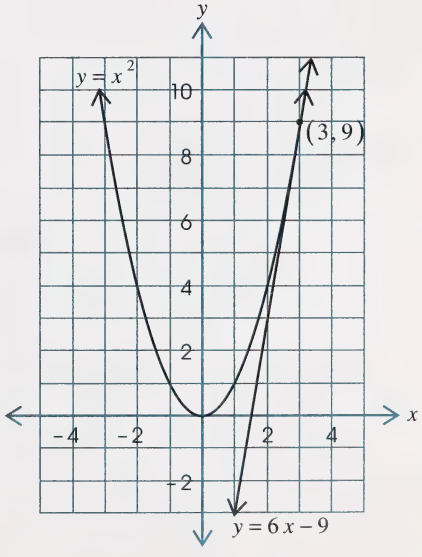
$$\therefore\ x - \frac{1}{5} = \frac{\sqrt{6}}{5} \quad \text{or} \quad x - \frac{1}{5} = \frac{\sqrt{6}}{5}$$
$$x = \frac{1}{5} + \frac{\sqrt{6}}{5} \qquad x = \frac{1}{5} - \frac{\sqrt{6}}{5}$$
$$\doteq 0.69 \qquad \doteq -0.29$$

The roots are $\frac{1}{5} + \frac{\sqrt{6}}{5}$ and $\frac{1}{5} - \frac{\sqrt{6}}{5}$, or approximately 0.69 and -0.29.

Enrichment

1. Graph the following system of equations:

$$y = x^2 \quad (1)$$
$$y = 6x - 9 \quad (2)$$



The line $y = 6x - 9$ appears to be tangent to $y = x^2$ at $(3, 9)$.

Therefore, the solution to $x^2 - 6x + 9 = 0$ appears to be $x = 3$.

Check

LS	RS
$x^2 - 6x + 9$	0
$= 3^2 - 6(3) + 9$	
$= 9 - 18 + 9$	
$= 0$	

LS = RS

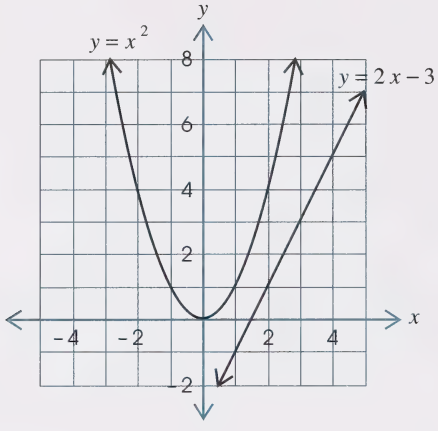
Enrichment (continued)

Notice that when the original quadratic equation is solved by factoring, the roots are real and equal.

$$x^2 - 6x + 9 = 0$$
$$(x-3)^2 = 0$$
$$x - 3 = 0$$
$$x = 3$$

2. Graph the following system of equations:

$y = x^2$ (1)
$y = 2x - 3$ (2)



The graphs do not intersect. Therefore, the equation $x^2 = 2x - 3$ has no real solution.

Section 3: Activity 1

1. a. Textbook question "Explore: Describe the Steps," p. 174

Step 1: Divide both sides of the equation by a.

Step 2: Subtract $\frac{c}{a}$ from both sides of the equation.

Step 3: Complete the square by adding the following to both sides:

$$\left(\frac{b}{a} \div 2\right)^2 = \left(\frac{b}{2a}\right)^2$$
$$= \frac{b^2}{4a^2}$$

Step 4: Express the left side as the square of a binomial.

Step 5: Combine $\frac{b^2}{4a^2} - \frac{c}{a}$ using a common denominator of $4a^2$.

Step 6: Calculate the square roots of both sides.

Step 7: Equate $\sqrt{4a^2}$ in the denominator on the right side to $2a$.

Step 8: Subtract $\frac{b}{2a}$ from both sides.

Step 9: Replace $\frac{b}{2a} - \frac{b}{2a}$ by 0.

Step 10: Rewrite the expression using a common denominator of $2a$.

b. Textbook questions 1 to 4 of "Inquire," p. 175

1. $a = 1$, $b = 3$, and $c = 2$

2. a. $b^2 - 4ac = 3^2 - 4(1)(2)$
$= 9 - 8$
$= 1$

b. $\sqrt{b^2 - 4ac} = \sqrt{1}$
$= 1$

c. $\frac{-b + \sqrt{b^2 - 4ac}}{2a} = \frac{-3+1}{2(1)}$
$= \frac{-2}{2}$
$= -1$

d. $\frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-3-1}{2(1)}$
$= \frac{-4}{2}$
$= -2$

3. For $x = -1$,

LS	RS
$x^2 + 3x + 2$	0
$= (-1)^2 + 3(-1) + 2$	
$= 1 - 3 + 2$	
$= 0$	

LS = RS

For $x = -2$,

LS	RS
$x^2 + 3x + 2$	0
$= (-2)^2 + 3(-2) + 2$	
$= 4 - 6 + 2$	
$= 0$	

LS = RS

Section 3: Activity 1 (continued)

4. a. $x^2+4x+3=0$

$\therefore\ a=1,\ b=4,$ and $c=3$

Substitute these values into the quadratic formula.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-4\pm\sqrt{4^2-4(1)(3)}}{2(1)}$$
$$=\frac{-4\pm\sqrt{16-12}}{2}$$
$$=\frac{-4\pm 2}{2}$$
$$=-2\pm 1$$

$\therefore\ x=-2+1$ or $x=-2-1$

$=-1$ $\quad$ $=-3$

Check

For $x=-1$,

LS	RS
x^2+4x+3	0
$=(-1)^2+4(-1)+3$	
$=1-4+3$	
$=0$	

LS = RS

For $x=-3$,

LS	RS
x^2+4x+3	0
$=(-3)^2+4(-3)+3$	
$=9-12+3$	
$=0$	

LS = RS

The roots are -1 and -3.

b. $x^2 + x - 12 = 0$

$\therefore\ a = 1,\ b = 1,$ and $c = -12$

Substitute these values into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-1 \pm \sqrt{1^2 - 4(1)(-12)}}{2(1)}$$
$$= \frac{-1 \pm \sqrt{1 + 48}}{2}$$
$$= \frac{-1 \pm \sqrt{49}}{2}$$
$$= \frac{-1 \pm 7}{2}$$

$\therefore\ x = \dfrac{-1+7}{2}$ or $x = \dfrac{-1-7}{2}$

$= \dfrac{6}{2}$ $= \dfrac{-8}{2}$

$= 3$ $= -4$

Check

For $x = 3$,

LS	RS
$x^2 + x - 12$	0
$= 3^2 + 3 - 12$	
$= 9 + 3 - 12$	
$= 0$	

LS = RS

For $x = -4$,

LS	RS
$x^2 + x - 12$	0
$= (-4)^2 + (-4) - 12$	
$= 16 - 4 - 12$	
$= 0$	

LS = RS

The roots are 3 and -4.

Section 3: Activity 1 (continued)

c. $x^2 + 4x - 5 = 0$

$\therefore\ a = 1,\ b = 4,$ and $c = -5$

Substitute these values into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-4 \pm \sqrt{4^2 - 4(1)(-5)}}{2(1)}$$
$$= \frac{-4 \pm \sqrt{16 + 20}}{2}$$
$$= \frac{-4 \pm \sqrt{36}}{2}$$
$$= \frac{-4 \pm 6}{2}$$
$$= -2 \pm 3$$

$\therefore\ x = -2 + 3$ or $x = -2 - 3$

$= 1$ $= -5$

Check

For $x = 1$,

LS	RS
$x^2 + 4x - 5$	0
$= 1^2 + 4(1) - 5$	
$= 1 + 4 - 5$	
$= 0$	

LS = RS

For $x = -5$,

LS	RS
$x^2 + 4x - 5$	0
$= (-5)^2 + 4(-5) - 5$	
$= 25 - 20 - 5$	
$= 0$	

LS = RS

The roots are 1 and -5.

2. a. Textbook questions 1, 4, 6, 9, 17, 23, 30, 34, 39, 46, 50, 54, 56, and 58 of "Practice," pp. 178 and 179

1. $x^2 + 6x + 5 = 0$

$\therefore\ a = 1,\ b = 6,$ and $c = 5$

Substitute these values into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-6 \pm \sqrt{6^2 - 4(1)(5)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{36 - 20}}{2}$$

$$= \frac{-6 \pm \sqrt{16}}{2}$$

$$= \frac{-6 \pm 4}{2}$$

$$= -3 \pm 2$$

$\therefore\ x = -3 + 2$ or $x = -3 - 2$

$= -1$ $= -5$

The roots are -1 and -5.

Section 3: Activity 1 (continued)

4. $x^2 - 12x + 35 = 0$

$\therefore\ a = 1,\ b = -12,$ and $c = 35$

Substitute these values into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(35)}}{2(1)}$$

$$= \frac{12 \pm \sqrt{144 - 140}}{2}$$

$$= \frac{12 \pm \sqrt{4}}{2}$$

$$= \frac{12 \pm 2}{2}$$

$$= 6 \pm 1$$

$\therefore\ x = 6 + 1$ or $x = 6 - 1$

$= 7$ $= 5$

The roots are 7 and 5.

6. $y^2 - 2y + 1 = 0$

$\therefore\ a = 1,\ b = -2,$ and $c = 1$

Substitute these values into the quadratic formula.

$$\begin{aligned} y &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(1)}}{2(1)} \\ &= \frac{2 \pm \sqrt{4-4}}{2} \\ &= \frac{2 \pm 0}{2} \\ &= \frac{2}{2} \\ &= 1 \end{aligned}$$

The root is 1.

9. $2x^2 - 5x - 12 = 0$

$\therefore\ a = 2,\ b = -5,$ and $c = -12$

Substitute these values into the quadratic formula.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-12)}}{2(2)} \\ &= \frac{5 \pm \sqrt{25 + 96}}{4} \\ &= \frac{5 \pm \sqrt{121}}{4} \\ &= \frac{5 \pm 11}{4} \end{aligned}$$

Section 3: Activity 1 (continued)

$$\therefore\ x = \frac{5+11}{4} \quad \text{or} \quad x = \frac{5-11}{4}$$

$$= \frac{16}{4} \qquad = \frac{-6}{4}$$

$$= 4 \qquad = -\frac{3}{2}$$

Check

For $x = 4$,

LS	RS
$2x^2 - 5x - 12$	0
$= 2(4)^2 - 5(4) - 12$	
$= 32 - 20 - 12$	
$= 0$	

LS = RS

For $x = -\frac{3}{2}$,

LS	RS
$2x^2 - 5x - 12$	0
$= 2\left(-\frac{3}{2}\right)^2 - 5\left(-\frac{3}{2}\right) - 12$	
$= \cancel{2}^{1}\left(\frac{9}{\cancel{4}_{2}}\right) + \frac{15}{2} - 12$	
$= \frac{9}{2} + \frac{15}{2} - 12$	
$= 0$	

LS = RS

The roots are 4 and $-\frac{3}{2}$.

17. $x^2+4x+2=0$

$\therefore\ a=1,\ b=4,$ and $c=2$

Substitute these values into the quadratic formula.

$$\begin{aligned}
x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a} \\
&= \frac{-4\pm\sqrt{4^2-4(1)(2)}}{2(1)} \\
&= \frac{-4\pm\sqrt{16-8}}{2} \\
&= \frac{-4\pm\sqrt{8}}{2} \\
&= \frac{-4\pm 2\sqrt{2}}{2} \\
&= \frac{\cancel{2}\left(-2\pm\sqrt{2}\right)}{\cancel{2}} \\
&= -2\pm\sqrt{2}
\end{aligned}$$

Check

For $x=-2+\sqrt{2}$,

LS	RS
x^2+4x+1	0
$=\left(-2+\sqrt{2}\right)^2+4\left(-2+\sqrt{2}\right)+2$	
$=4-4\sqrt{2}+2-8+4\sqrt{2}+2$	
$=0$	

LS = RS

Section 3: Activity 1 (continued)

For $x = -2 - \sqrt{2}$,

	LS	RS
$x^2 + 4x + 1$		0
$= \left(-2-\sqrt{2}\right)^2 + 4\left(-2-\sqrt{2}\right) + 2$		
$= 4 + 4\sqrt{2} + 2 - 8 - 4\sqrt{2} + 2$		
$= 0$		
	LS =	RS

The roots are $-2+\sqrt{2}$ and $-2-\sqrt{2}$.

23. $7x^2 - 2x - 2 = 0$

$\therefore\ a = 7,\ b = -2,$ and $c = -2$

Substitute these values into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(7)(-2)}}{2(7)}$$

$$= \frac{2 \pm \sqrt{4 + 56}}{14}$$

$$= \frac{2 \pm \sqrt{60}}{14}$$

$$= \frac{2 \pm 2\sqrt{15}}{14}$$

$$= \frac{\overset{1}{\cancel{2}}\left(1 \pm \sqrt{15}\right)}{\underset{7}{\cancel{14}}}$$

$$= \frac{1 \pm \sqrt{15}}{7}$$

$$\therefore\ x = \frac{1+\sqrt{15}}{7} \quad \text{or} \quad x = \frac{1-\sqrt{15}}{7}$$

The roots are $\frac{1+\sqrt{15}}{7}$ and $\frac{1-\sqrt{15}}{7}$.

30. $0 = -3x^2 + 4x + 1$

$3x^2 - 4x - 1 = 0$ ← Rewrite with a positive value of a.

$\therefore\ a = 3,\ b = -4,$ and $c = -1$

Substitute these values into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-1)}}{2(3)}$$

$$= \frac{4 \pm \sqrt{16 + 12}}{6}$$

$$= \frac{4 \pm \sqrt{28}}{6}$$

$$= \frac{4 \pm 2\sqrt{7}}{6}$$

$$= \frac{\overset{1}{\cancel{2}}\left(2 \pm \sqrt{7}\right)}{\underset{3}{\cancel{6}}}$$

$$= \frac{2 \pm \sqrt{7}}{3}$$

$$\therefore\ x = \frac{2 + \sqrt{7}}{3} \quad \text{or} \quad x = \frac{2 - \sqrt{7}}{3}$$

The roots are $\frac{2+\sqrt{7}}{3}$ and $\frac{2-\sqrt{7}}{3}$.

Section 3: Activity 1 (continued)

34. $2c(c-3)=7$

$2c^2-6c=7$

$2c^2-6c-7=0$ ← Express in standard form.

$\therefore\ a=2,\ b=-6,$ and $c=-7$

↑ This is a coefficient, not the variable c in the original equation.

Substitute these values into the quadratic formula.

$$c=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-6)\pm\sqrt{(-6)^2-4(2)(-7)}}{2(2)}$$

$$=\frac{6\pm\sqrt{36+56}}{4}$$

$$=\frac{6\pm\sqrt{92}}{4}$$

$$=\frac{6\pm2\sqrt{23}}{4}$$

$$=\frac{\overset{1}{\cancel{2}}\left(3\pm\sqrt{23}\right)}{\underset{2}{\cancel{4}}}$$

$$=\frac{3\pm\sqrt{23}}{2}$$

$$\therefore\ c=\frac{3+\sqrt{23}}{2} \quad \text{or} \quad c=\frac{3-\sqrt{23}}{2}$$

$$\doteq 3.9 \qquad \doteq -0.9$$

The roots are approximately 3.9 and -0.9.

39. $(2d+3)(d-2)=(d+9)(d-3)+16$

$2d^2-4d+3d-6=d^2-3d+9d-27+16$

$2d^2-d-6=d^2+6d-11$

$d^2-7d+5=0$

$\therefore\ a=1,\ b=-7,$ and $c=5$

Substitute these values into the quadratic formula.

$$d=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-7)\pm\sqrt{(-7)^2-4(1)(5)}}{2(1)}$$

$$=\frac{7\pm\sqrt{49-20}}{2}$$

$$=\frac{7\pm\sqrt{29}}{2}$$

$$\therefore\ d=\frac{7+\sqrt{29}}{2} \quad \text{or} \quad d=\frac{7-\sqrt{29}}{2}$$

$$\doteq 6.2 \qquad \doteq 0.8$$

The roots are approximately 6.2 and 0.8.

46. $0.04a^2+0.1a+0.05=5$

$4a^2+10a+5=500$ ← Multiply both sides by 100.

$4a^2+10a-495=0$

This is a coefficient, not the variable a in the original equation.

$\therefore\ a=4,\ b=10,$ and $c=-495$

Substitute these values into the quadratic formula.

$$a=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-10\pm\sqrt{10^2-4(4)(-495)}}{2(4)}$$

$$=\frac{-10\pm\sqrt{100+7920}}{8}$$

$$=\frac{-10\pm\sqrt{8020}}{8}$$

$$\therefore\ a=\frac{-10+\sqrt{8020}}{8} \quad \text{or} \quad a=\frac{-10-\sqrt{8020}}{8}$$

$$\doteq 9.94 \qquad \doteq -12.44$$

The roots are approximately 9.94 and -12.44.

Section 3: Activity 1 (continued)

50. $\dfrac{2}{x-3}+\dfrac{3}{x+2}=1$

$2(x+2)+3(x-3)=1(x-3)(x+2)$ ← Multiply both sides by $(x-3)(x+2)$.

$2x+4+3x-9=x^2+2x-3x-6$

$5x-5=x^2-x-6$

$0=x^2-6x-1$

$x^2-6x-1=0$

$\therefore\ a=1,\ b=-6,$ and $c=-1$

Substitute these values into the quadratic formula.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(-1)}}{2(1)}$$

$$=\frac{6\pm\sqrt{36+4}}{2}$$

$$=\frac{6\pm\sqrt{40}}{2}$$

$$=\frac{6\pm2\sqrt{10}}{2}$$

$$=\frac{\cancel{2}\left(3\pm\sqrt{10}\right)}{\cancel{2}}$$

$$=3\pm\sqrt{10}$$

$\therefore\ x=3+\sqrt{10}$ or $x=3-\sqrt{10}$

$\doteq 6.16$ $\doteq -0.16$

The roots are approximately 6.16 and -0.16.

54. $\dfrac{x-3}{x+4}=\dfrac{x+3}{2x-1}$

$(x-3)(2x-1)=(x+3)(x+4)$ ← Multiply both sides by $(x+4)(2x-1)$.

$2x^2-x-6x+3=x^2+4x+3x+12$

$2x^2-7x+3=x^2+7x+12$

$x^2-14x-9=0$

$\therefore\ a=1,\ b=-14,$ and $c=-9$

Substitute these values into the quadratic formula.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-14)\pm\sqrt{(-14)^2-4(1)(-9)}}{2(1)}$$

$$=\frac{14\pm\sqrt{196+36}}{2}$$

$$=\frac{14\pm\sqrt{232}}{2}$$

$$=\frac{14\pm2\sqrt{58}}{2}$$

$$=\frac{2\left(7\pm\sqrt{58}\right)}{2}$$

$$=7\pm\sqrt{58}$$

$\therefore\ x=7+\sqrt{58}$ or $x=7-\sqrt{58}$

The roots are $7+\sqrt{58}$ and $7-\sqrt{58}$.

Section 3: Activity 1 (continued)

56. $\dfrac{2}{x-1}-\dfrac{1}{x+1}=\dfrac{x^2}{x^2-1}$

$\dfrac{2}{x-1}-\dfrac{1}{x+1}=\dfrac{x^2}{(x-1)(x+1)}$ ← Multiply both sides by $(x-1)(x+1)$.

$2(x+1)-(x-1)=x^2$

$2x+2-x+1=x^2$

$x+3=x^2$

$0=x^2-x-3$

$x^2-x-3=0$

$\therefore\ a=1,\ b=-1,$ and $c=-3$

Substitute these values into the quadratic formula.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-3)}}{2(1)}$$

$$=\frac{1\pm\sqrt{1+12}}{2}$$

$$=\frac{1\pm\sqrt{13}}{2}$$

$\therefore\ x=\dfrac{1+\sqrt{13}}{2}$ and $x=\dfrac{1-\sqrt{13}}{2}$.

The roots are $\frac{1+\sqrt{13}}{2}$ and $\frac{1-\sqrt{13}}{2}$.

58.

$$\frac{2x+1}{x^2-4x+3}-\frac{3x+2}{x^2+x-2}=\frac{4}{x^2-x-6}$$

$$\frac{2x+1}{(x-1)(x-3)}-\frac{3x+2}{(x-1)(x+2)}=\frac{4}{(x-3)(x+2)}$$ ← Multiply both sides by $(x-1)(x-3)(x+2)$.

$$(2x+1)(x+2)-(3x+2)(x-3)=4(x-1)$$

$$\left(2x^2+4x+x+2\right)-\left(3x^2-9x+2x-6\right)=4x-4$$

$$2x^2+5x+2-3x^2+7x+6=4x-4$$

$$-x^2+12x+8=4x-4$$

$$-x^2+8x+12=0$$

$$x^2-8x-12=0$$

$\therefore\ a=1,\ b=-8,$ and $c=-12$

Substitute these values into the quadratic formula.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-8)\pm\sqrt{(-8)^2-4(1)(-12)}}{2(1)}$$

$$=\frac{8\pm\sqrt{64+48}}{2}$$

$$=\frac{8\pm\sqrt{112}}{2}$$

$$=\frac{8\pm4\sqrt{7}}{2}$$

$$=\frac{2\left(4\pm2\sqrt{7}\right)}{2}$$

$$=4\pm2\sqrt{7}$$

$\therefore\ x=4+2\sqrt{7}$ or $x=4-2\sqrt{7}$

The roots are $4+2\sqrt{7}$ and $4-2\sqrt{7}$.

Section 3: Activity 1 (continued)

b. Textbook questions 59.b., 63, 64, 66, 72, 77.a., 81, and 85.b. of "Applications and Problem Solving," pp. 179 and 180

59. b. $-4.9t^2 - 5t + 90 = 0$

$4.9t^2 + 5t - 90 = 0$ ← Multiply both sides by -1.

$\therefore a = 4.9,\ b = 5,$ and $c = -90$

Substitute these values into the quadratic formula.

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{5^2 - 4(4.9)(-90)}}{2(4.9)}$$

$$= \frac{-5 \pm \sqrt{25 + 1764}}{9.8}$$

$$= \frac{-5 \pm \sqrt{1789}}{9.8}$$

$$\therefore t = \frac{-5 + \sqrt{1789}}{9.8} \quad \text{or} \quad t = \frac{-5 - \sqrt{1789}}{9.8}$$

$$\doteq 3.8 \qquad \doteq -4.8$$

Since time is non-negative, the object takes approximately 3.8 s to fall to the ground.

63. Let x be the width, and let $x+60$ be the length.

$$\therefore \text{Area} = \text{length} \times \text{width}$$
$$6496 = x(x+60)$$
$$x(x+60) = 6496$$
$$x^2 + 60x - 6496 = 0$$

$\therefore\ a=1,\ b=60,$ and $c=-6496$

Substitute these values into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-60 \pm \sqrt{60^2 - 4(1)(-6496)}}{2(1)}$$
$$= \frac{-60 \pm \sqrt{3600 + 25\,984}}{2}$$
$$= \frac{-60 \pm \sqrt{29\,584}}{2}$$
$$= \frac{-60 \pm 172}{2}$$
$$= -30 \pm 86$$

$\therefore\ x = -30+86$ or $x = -30-86$
$= 56$ $= -116$

Because width is non-negative, the width is 56 m and the length is $56+60=116$ m.

Section 3: Activity 1 (continued)

64. Let x be the speed from Glasgow to Halifax, and let $x+100$ be the speed from Halifax to Glasgow.

Trip	Distance (km)	Speed (km/h)	Time (h)
Glasgow to Halifax	4200	x	$\frac{4200}{x}$
Halifax to Glasgow	4200	$x+100$	$\frac{4200}{x+100}$

$$\text{Total time} = \frac{4200}{x} + \frac{4200}{x+100}$$

$$13 = \frac{4200}{x} + \frac{4200}{x+100}$$

$$13x(x+100) = 4200(x+100) + 4200x$$

$$13x^2 + 1300x = 4200x + 420\,000 + 4200x$$

$$13x^2 + 1300x = 8400x + 420\,000$$

$$13x^2 - 7100x - 420\,000 = 0$$

$\therefore\ a = 13,\ b = -7100,$ and $c = -420\,000$

Substitute these values into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-7100) \pm \sqrt{(-7100)^2 - 4(13)(-420\,000)}}{2(13)}$$

$$= \frac{7100 \pm \sqrt{72\,250\,000}}{26}$$

$$= \frac{7100 \pm 8500}{26}$$

$$\therefore\ x = \frac{7100 + 8500}{26} \quad \text{or} \quad x = \frac{7100 + 8500}{26}$$

$$= \frac{15\,600}{600} \qquad = \frac{-1400}{26}$$

$$= 600 \qquad = -\frac{700}{13}$$

Because speed is non-negative, the flying speed from Glasgow to Halifax was 600 km/h.

66. Let x be the average speed from Winnipeg to Billings, and let $x+5$ be the average speed from Billings to Winnipeg.

Trip	Distance (km)	Speed (km/h)	Time (h)
Winnipeg to Billings	1200	x	$\frac{1200}{x}$
Billings to Winnipeg	1200	$x+5$	$\frac{1200}{x+5}$

$$\text{Total time} = \frac{1200}{x} + \frac{1200}{x+5}$$

$$31 = \frac{1200}{x} + \frac{1200}{x+5}$$

$$31x(x+5) = 1200(x+5) + 1200x$$

$$31x^2 + 155x = 1200x + 6000 + 1200x$$

$$31x^2 + 155x = 2400x + 6000$$

$$31x^2 - 2245x - 6000 = 0$$

$\therefore a = 31$, $b = -2245$, and $c = -6000$

Substitute these values into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2245) \pm \sqrt{(-2245)^2 - 4(31)(-6000)}}{2(31)}$$

$$= \frac{2245 \pm \sqrt{5\,784\,025}}{62}$$

$$= \frac{2245 \pm 2405}{62}$$

$$\therefore x = \frac{2245 + 2405}{62} \quad \text{or} \quad x = \frac{2245 - 2405}{62}$$

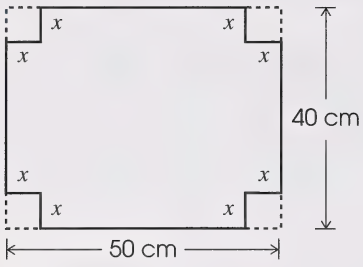
$$= \frac{4650}{62} \qquad = \frac{-160}{62}$$

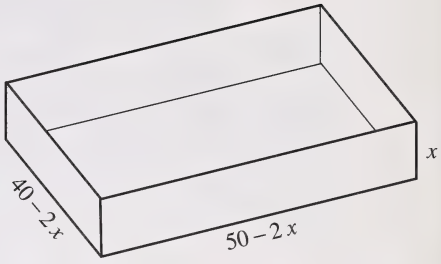
$$= 75 \qquad \doteq -2.58$$

Because speed is non-negative, the average speed from Winnipeg to Billings was 75 km/h.

Section 3: Activity 1 (continued)

72. a. Let x be the side length of each removed square.





$$\text{Area of base} = (40-2x)(50-2x)$$

$$875 = 2000 - 80x - 100x + 4x^2$$

$$875 = 2000 - 180x + 4x^2$$

$$4x^2 - 180x + 1125 = 0$$

$\therefore\ a = 4,\ b = -180,$ and $c = 1125$

Substitute these values into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-180) \pm \sqrt{(-180)^2 - 4(4)(1125)}}{2(4)}$$

$$= \frac{180 \pm \sqrt{14\,400}}{8}$$

$$= \frac{180 \pm 120}{8}$$

$$\therefore\ x = \frac{180+120}{8} \quad \text{or} \quad x = \frac{180-120}{8}$$

$$= 37.5 \qquad\qquad = 7.5$$

The largest value of x is $\frac{1}{2}(40) = 20\,\text{cm}$, because the shorter side of the original cardboard rectangle was 40 cm.

$\therefore\ x = 7.5\,\text{cm}$

Squares of 7.5 cm on a side should be removed.

b. Volume = length × width × height

$$\begin{aligned}
&= (50-2x)(40-2x)(x) \\
&= [50-2(7.5)][40-2(7.5)](7.5) \\
&= (50-15)(40-15)(7.5) \\
&= (35)(25)(7.5) \\
&= 6562.5
\end{aligned}$$

The volume of the box is 6562.5 cm^3.

77. a. Let x be the number of \$10 decreases in price; therefore, the price of a jacket is $200-10x$ and the number of jackets sold is $90+5x$.

$$\begin{aligned}
\therefore \text{Revenue} &= \text{Price} \times \text{Number of jackets sold} \\
17\,600 &= (200-10x)(90+5x) \\
17\,600 &= 18\,000 + 1000x - 900x - 50x^2 \\
17\,600 &= -50x^2 + 100x + 18\,000 \\
50x^2 - 100x - 400 &= 0 \\
x^2 - 2x - 8 &= 0
\end{aligned}$$

$\therefore\ a=1,\ b=-2,$ and $c=-8$

Substitute these values into the quadratic formula.

$$\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2-4ac}}{2a} \\
&= \frac{-(-2) \pm \sqrt{(-2)^2-4(1)(-8)}}{2(1)} \\
&= \frac{2 \pm \sqrt{4+32}}{2} \\
&= \frac{2 \pm 6}{2} \\
&= 1 \pm 3
\end{aligned}$$

$\therefore\ x = 1+3 \quad \text{or} \quad x = 1-3$

$\quad = 4 \qquad\qquad = -2$

Section 3: Activity 1 (continued)

If $x = 4$, each jacket would sell for $200 - 10(4)$ or \$160. The number sold would be $90 + 5(4) = 110$.

$\therefore$ Revenue $= 160 \times 110$
$= \$17\,600$

If $x = -2$, each jacket would sell for $200 - 10(-2) = \$220$. The number sold would be $90 + 5(-2) = 80$.

$\therefore$ Revenue $= 220 \times 80$
$= \$17\,600$

81. If a quadratic equation can be solved easily by factoring, the roots must be rational. The expression $b^2 - 4ac$ must be a perfect square, otherwise $\sqrt{b^2 - 4ac}$ would be irrational.

85. b. $x = 3 + \sqrt{5}$ or $x = 3 - \sqrt{5}$

$x - 3 - \sqrt{5} = 0$ $\qquad$ $x - 3 + \sqrt{5} = 0$

$$\therefore \left(x - 3 - \sqrt{5}\right)\left(x - 3 + \sqrt{5}\right) = 0$$

$$x\left(x - 3 + \sqrt{5}\right) - 3\left(x - 3 + \sqrt{5}\right) - \sqrt{5}\left(x - 3 + \sqrt{5}\right) = 0$$

$$x^2 - 3x + \cancel{\sqrt{5}x} - 3x + 9 - \cancel{3\sqrt{5}} - \cancel{\sqrt{5}x} + \cancel{3\sqrt{5}} - 5 = 0$$

$$x^2 - 6x + 4 = 0$$

A quadratic equation with roots $3 + \sqrt{5}$ and $3 - \sqrt{5}$ is $x^2 - 6x + 4 = 0$.

3. a. Textbook questions 1 to 4 of Investigation 1, "The Sum of Consecutive Squares," p. 150

1. Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

Add the squares of two consecutive terms.

$1^2 + 1^2 = 2 \longleftrightarrow t_1^2 + t_2^2 = t_3$

$1^2 + 2^2 = 5 \longleftrightarrow t_2^2 + t_3^2 = t_5$

$2^2 + 3^2 = 13 \longleftrightarrow t_3^2 + t_4^2 = t_7$

$3^2 + 5^2 = 34 \longleftrightarrow t_4^2 + t_5^2 = t_9$

2. The subscript of the result is the sum of the subscripts of the terms whose squares are added.

3. **a.** $t_9{}^2 + t_{10}{}^2 = t_{19}$ **b.** $(t_{21})^2 + (t_{22})^2 = t_{43}$

4. $(t_n)^2 + (t_{n+1})^2 = t_{n+(n+1)}$
$= t_{2n+1}$

b. Textbook questions 1 to 3 of Investigation 2, "The Difference Between Alternate Squares," p. 150

1. Fibonacci Sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, …

$t_3{}^2 - t_1{}^2 = 2^2 - 1^2$
$= 3$
$= t_4$

$t_4{}^2 - t_2{}^2 = 3^2 - 1^2$
$= 8$
$= t_6$

$t_5{}^2 - t_3{}^2 = 5^2 - 2^2$
$= 21$
$= t_8$

$t_6{}^2 - t_4{}^2 = 8^2 - 3^2$
$= 64 - 9$
$= 55$
$= t_{10}$

2. The subscript of the result is the sum of the subscripts of the alternate terms whose squares are subtracted.

3. $(t_{n+2})^2 - (t_n)^2 = t_{(n+2)+n}$
$= t_{2n+2}$

Note: The statement given in the textbook is incorrect. It should be written as a difference, not a sum.

c. Textbook questions 1 and 2 of Investigation 3, "Four Consecutive Fibonacci Numbers," p. 150

1. Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, …

Four Consecutive Terms	Difference of Squares of Middle 2 Terms	Product of the First and Last Terms
1, 1, 2, 3	$2^2 - 1^2 = 3$	$1 \times 3 = 3$
1, 2, 3, 5	$3^2 - 2^2 = 5$	$1 \times 5 = 5$
2, 3, 5, 8	$5^2 - 3^2 = 16$	$2 \times 8 = 16$
3, 5, 8, 13	$8^2 - 5^2 = 39$	$3 \times 13 = 39$

Section 3: Activity 1 (continued)

2. The difference of the squares of the middle two terms, of a sequence of four consecutive terms taken from the Fibonacci sequence, is equal to the product of the first and fourth terms. That is, for t_n, t_{n+1}, t_{n+2}, t_{n+3}, the relationship is $(t_{n+2})^2 - (t_{n+1})^2 = t_n \bullet t_{n+3}$.

Section 3: Activity 2

1. a. Textbook questions a. to e. of "Explore: Use the Definitions," p. 181

a. $\sqrt{-3} = \sqrt{-1} \times \sqrt{3}$
$= i\sqrt{3}$

b. $\sqrt{-9} = \sqrt{-1} \times \sqrt{9}$
$= i \times 3$
$= 3i$

c. $\sqrt{-16} = \sqrt{-1} \times \sqrt{16}$
$= i \times 4$
$= 4i$

d. $\sqrt{-8} = \sqrt{-1} \times \sqrt{8}$
$= i \times 2\sqrt{2}$
$= 2i\sqrt{2}$

e. $\sqrt{-20} = \sqrt{-1} \times \sqrt{20}$
$= i \times 2\sqrt{5}$
$= 2i\sqrt{5}$

b. Textbook questions 1 to 3 of "Inquire," p. 181

1. To find the square root of a negative number, express the root as the product of $\sqrt{-1}$ and the root of a positive number. Replace $\sqrt{-1}$ by i, and simplify the root of the positive number.

2. a. $\sqrt{-10} = \sqrt{-1} \times \sqrt{10}$
$= i\sqrt{10}$

b. $\sqrt{-18} = \sqrt{-1} \times \sqrt{18}$
$= i \times 3\sqrt{2}$
$= 3i\sqrt{2}$

c. $\sqrt{-28} = \sqrt{-1} \times \sqrt{28}$
$= i \times 2\sqrt{7}$
$= 2i\sqrt{7}$

d. $\sqrt{-50} = \sqrt{-1} \times \sqrt{50}$
$= i \times 5\sqrt{2}$
$= 5i\sqrt{2}$

3. a. $5i \times 5i = 25i^2$
$= 25(-1)$
$= -25$

b. $2i \times 3i = 6i^2$
$= 6(-1)$
$= -6$

c. $(-2i)\times(-2i)=4i^2$
$=4(-1)$
$=-4$

d. $(-3i)\times(-4i)=12i^2$
$=12(-1)$
$=-12$

e. $2i\times(-5i)=-10i^2$
$=-10(-1)$
$=10$

f. $(-3i)\times 6i=-18i^2$
$=-18(-1)$
$=18$

2. a. Textbook questions 1, 4, 7, 10, 16, 19 to 22, 29, 32, 34, 36, 44, 46, 48, 51, 53, and 54 of "Practice," pp. 185 and 186

1. $\sqrt{-9}=\sqrt{-1}\left(\sqrt{9}\right)$
$=i(3)$
$=3i$

4. $\sqrt{-5}=\sqrt{-1}\left(\sqrt{5}\right)$
$=i\sqrt{5}$

7. $\sqrt{-12}=\sqrt{-1}\sqrt{12}$
$=i\left(2\sqrt{3}\right)$
$=2i\sqrt{3}$

10. $-\sqrt{-4}=-\sqrt{-1}\left(\sqrt{4}\right)$
$=-i\bullet 2$
$=-2i$

16. $\left(\sqrt{-6}\right)^2=\left(i\sqrt{6}\right)^2$
$=i^2\left(\sqrt{6}\right)^2$
$=-1(6)$
$=-6$

19. $i^3=i^2\bullet i$
$=-1(i)$
$=-i$

20. $i^4=i^2\left(i^2\right)$
$=-1(-1)$
$=1$

21. $i^5=i^4\,(i)$
$=1i$
$=i$

22. $4i\times 5i=20i^2$
$=20(-1)$
$=-20$

29. $-\left(i\sqrt{5}\right)^2=-\left(i^2\right)(5)$
$=-(-1)(5)$
$=5$

Section 3: Activity 2 (continued)

32. $\left(-5i\sqrt{2}\right)^2 = (-5)^2\left(i^2\right)\left(\sqrt{2}\right)^2$
$= 25(-1)(2)$
$= -50$

34. $(4+2i)+(3-4i) = 4+2i+3-4i$
$= 7-2i$

36. $(3-2i)-(1+3i) = 3-2i-1-3i$
$= 2-5i$

44. $2(4-3i) = 8-6i$

46. $-4i(3-5i) = -12i+20i^2$
$= -12i+20(-1)$
$= -12i-20$
$= -20-12i$

48. $(2-4i)(1+3i) = 2+6i-4i-12i^2$
$= 2+2i-12(-1)$
$= 2+2i+12$
$= 14+2i$

51. $(1-5i)(1+5i) = 1-25i^2$
$= 1-25(-1)$
$= 1+25$
$= 26$

53. $(4i-3)^2 = 16i^2-24i+9$
$= 16(-1)-24i+9$
$= -16-24i+9$
$= -7-24i$

54. $(i-1)^2 = i^2-2i+1$
$= -1-2i+1$
$= -2i$

b. Textbook questions 88, 89.a., 91.a., and 91.c. of "Applications and Problem Solving," p. 186

88. a. $(a+bi)(a-bi) = a^2-b^2i^2$
$= a^2-b^2(-1)$
$= a^2+b^2$

Because a and b are real numbers, a^2+b^2 is a real number.

b. $(5+7i)(5-7i) = 25-49i^2$
$= 25-49(-1)$
$= 25+49$
$= 74$

c. $(3+4i)(3-4i) = 9-16i^2$
$= 9-16(-1)$
$= 9+16$
$= 25$

$(4+3i)(4-3i) = 16-9i^2$
$= 16-9(-1)$
$= 16+9$
$= 25$

89. a. $f(0) = 0^2 + i$
$= i$

$f(i) = i^2 + i$
$= -1+i$

$f(-1+i) = (-1+i)^2 + i$
$= 1-2i+i^2+i$
$= 1-i-1$
$= -i$

91. a. $6+12i = 2s+3ti$

Relate real parts.

$2s = 6$
$s = 3$

Relate imaginary parts.

$3ti = 12i$
$t = 4$

c. $s+3t+2si-ti = 5+3i$
$(s+3t)+(2s-t)i = 5+3i$

$s+3t = 5$ (1)
$2s-t = 3$ (2)

$s+3t = 5$ (1)
$6s-3t = 9$ (3) : $3\times$ (2)
$7s = 14$ (1) + (3)
$s = 2$

Substitute in (1).

$s+3t = 5$
$2+3t = 5$
$3t = 3$
$t = 1$

The values of s and t are 2 and 1, respectively.

Section 3: Activity 2 (continued)

3. a. Textbook questions 56, 61, 63, 68, 70, 75, 84, and 86 of "Practice," p. 186

56. $x^2+9=0$

$x^2=-9$

$x=\pm\sqrt{-9}$

$=\pm\sqrt{-1}\left(\sqrt{9}\right)$

$=\pm i(3)$

$=\pm 3i$

Check

For $x=3i$,

LS	RS
x^2+9	0
$=(3i)^2+9$	
$=9i^2+9$	
$=9(-1)+9$	
$=0$	

LS = RS

For $x=-3i$,

LS	RS
x^2+9	0
$=(-3i)^2+9$	
$=9i^2+9$	
$=9(-1)+9$	
$=0$	

LS = RS

The roots are $3i$ and $-3i$.

61. $-5x^2-90=0$

$-5x^2=90$

$x^2=-18$

$x=\pm\sqrt{-18}$

$=\pm\sqrt{-1}\left(\sqrt{18}\right)$

$=\pm i\left(3\sqrt{2}\right)$

$=\pm 3i\sqrt{2}$

Check

For $x = 3i\sqrt{2}$,

LS	RS
$-5x^2 - 90$	0
$= -5\left(3i\sqrt{2}\right)^2 - 90$	
$= -5\left(9i^2 \bullet 2\right) - 90$	
$= -5(-18) - 90$	
$= 90 - 90$	
$= 0$	

LS = RS

For $x = -3i\sqrt{2}$,

LS	RS
$-5x^2 - 90$	0
$= -5\left(-3i\sqrt{2}\right)^2 - 90$	
$= -5\left(9i^2 \bullet 2\right) - 90$	
$= -5(-18) - 90$	
$= 90 - 90$	
$= 0$	

LS = RS

The roots are $3i\sqrt{2}$ and $-3i\sqrt{2}$.

63. $x^2 - 4x + 8 = 0$

$\therefore\ a = 1,\ b = -4,$ and $c = 8$

Substitute these values into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(8)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 - 32}}{2}$$

$$= \frac{4 \pm \sqrt{-16}}{2}$$

$$= \frac{4 \pm 4i}{2}$$

$$= 2 \pm 2i$$

The roots are $2 + 2i$ and $2 - 2i$.

Section 3: Activity 2 (continued)

68. $2x^2+3x+3=0$

$\therefore\ a=2,\ b=3,$ and $c=3$

Substitute these values into the quadratic formula.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-3\pm\sqrt{3^2-4(2)(3)}}{2(2)}$$
$$=\frac{-3\pm\sqrt{9-24}}{4}$$
$$=\frac{-3\pm\sqrt{-15}}{4}$$
$$=\frac{-3\pm i\sqrt{15}}{4}$$

The roots are $\frac{-3+i\sqrt{15}}{4}$ and $\frac{-3-i\sqrt{15}}{4}$.

70. $5x^2+5x+2=0$

$\therefore\ a=5,\ b=5,$ and $c=2$

Substitute these values into the quadratic formula.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-5\pm\sqrt{5^2-4(5)(2)}}{2(5)}$$
$$=\frac{-5\pm\sqrt{25-40}}{10}$$
$$=\frac{-5\pm\sqrt{-15}}{10}$$
$$=\frac{-5\pm i\sqrt{15}}{10}$$

The roots are $\frac{-5+i\sqrt{15}}{10}$ and $\frac{-5-i\sqrt{15}}{10}$.

75. $$\frac{2}{x-2}+1=\frac{1}{x-1}$$

$$2(x-1)+1(x-2)(x-1)=1(x-2) \quad \longleftarrow \text{Multiply both sides by } (x-2)(x-1).$$

$$2x-2+\left(x^2-3x+2\right)=x-2$$

$$2x-2+x^2-3x+2=x-2$$

$$x^2-x=x-2$$

$$x^2-2x+2=0$$

$\therefore\ a=1,\ b=-2,$ and $c=2$

Substitute these values into the quadratic formula.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(2)}}{2(1)}$$

$$=\frac{2\pm\sqrt{4-8}}{2}$$

$$=\frac{2\pm\sqrt{-4}}{2}$$

$$=\frac{2\pm 2i}{2}$$

$$=1\pm i$$

The roots are $1+i$ and $1-i$.

Section 3: Activity 2 (continued)

84. $2x^4+5x^2+3=0$

$(2x^2+3)(x^2+1)=0$

$\therefore\ 2x^2+3=0$ or $x^2+1=0$

$2x^2=-3$ $\qquad$ $x^2=-1$

$x^2=\frac{-3}{2}$ $\qquad$ $x=\pm\sqrt{-1}$

$x=\pm\sqrt{\frac{-3}{2}}$ $\qquad$ $=\pm i$

$=\pm\frac{i\sqrt{3}}{\sqrt{2}}$

$=\pm\frac{i\sqrt{3}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$

$=\pm\frac{i\sqrt{6}}{2}$

The roots are $\frac{i\sqrt{6}}{2}$, $\frac{-i\sqrt{6}}{2}$, i, and $-i$.

86. $4a^4-1=0$

$4a^4=1$

$a^4=\frac{1}{4}$

$a^2=\pm\sqrt{\frac{1}{4}}$

$=\pm\frac{1}{2}$

$\therefore\ a^2=\frac{1}{2}$ or $a^2=-\frac{1}{2}$

$a=\pm\sqrt{\frac{1}{2}}$ $\qquad$ $a=\pm\sqrt{-\frac{1}{2}}$

$=\pm\frac{1}{\sqrt{2}}$ $\qquad$ $=\pm\frac{\sqrt{-1}}{\sqrt{2}}$

$=\pm\frac{1}{\sqrt{2}}\bullet\frac{\sqrt{2}}{\sqrt{2}}$ $\qquad$ $=\pm\frac{i}{\sqrt{2}}$

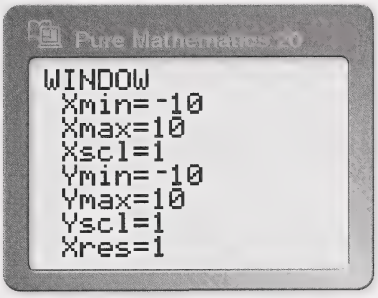
$=\pm\frac{\sqrt{2}}{2}$ $\qquad$ $=\pm\frac{i}{\sqrt{2}}\bullet\frac{\sqrt{2}}{\sqrt{2}}$

$=\pm\frac{i\sqrt{2}}{2}$

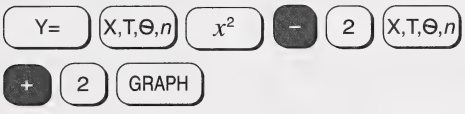
The roots are $\frac{\sqrt{2}}{2}$, $-\frac{\sqrt{2}}{2}$, $\frac{i\sqrt{2}}{2}$, and $-\frac{i\sqrt{2}}{2}$.

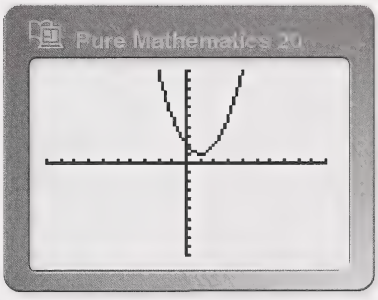
b. Textbook questions 92.a., 92.b., 93.b., and 95 of "Applications and Problem Solving," p. 186

92. a. Use the standard window settings.

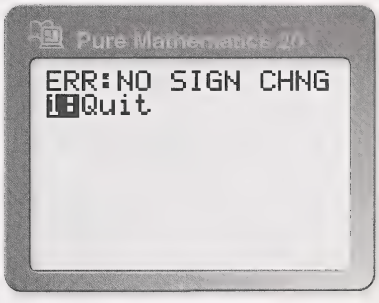


Now, graph the function, $y = x^2 - 2x + 2$.





b. When you try to find the zeros of the function using the zero operation, an error message occurs.



For a real zero to occur, there must be a sign change in the function from positive to negative or negative to positive. In other words, if $f(a) = 0$ for some real value of a, the graph of the function will cross the x-axis at $(a, 0)$. In the case of $y = x^2 - 2x + 2$, the graph is strictly above the x-axis. There is no sign change in $y = f(x)$. The function is always positively valued.

Section 3: Activity 2 (continued)

93. b. $x = 1+i$ and $x = 1-i$

$x-1-i=0$ $\quad$ $x-1+i=0$

$$\therefore (x-1-i)(x-1+i)=0$$
$$x(x-1+i)-1(x-1+i)-i(x-1+i)=0$$
$$x^2-x+ix-x+1-i-ix+i-i^2=0$$
$$x^2-2x+1-i^2=0$$
$$x^2-2x+1-(-1)=0$$
$$x^2-2x+1+1=0$$
$$x^2-2x+2=0$$

An equation with roots $1+i$ and $1-i$ is $x^2-2x+2=0$.

95. Yes, it is possible for a quadratic equation to have one real root and one imaginary root. For example, suppose $x=1$ and $x=i$ are roots.

$\therefore x=1$ and $x=i$

$x-1=0$ $\quad$ $x-i=0$

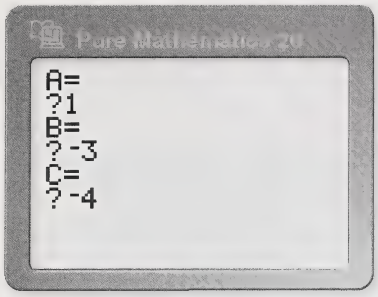
$$\therefore (x-1)(x-i)=0$$
$$x^2-ix-1x+i=0$$
$$x^2-(1+i)x+i=0$$

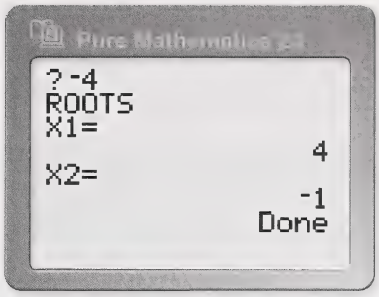
A quadratic equation with roots 1 and i is $x^2-(1+i)x+i=0$.

Notice that the coefficients of this equation are complex numbers.

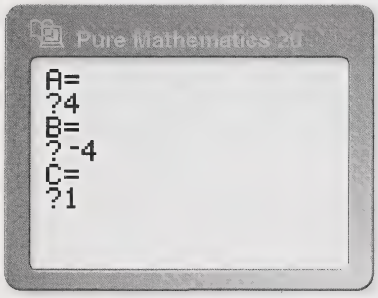
$a=1$, $b=-1-i$, and $c=i$

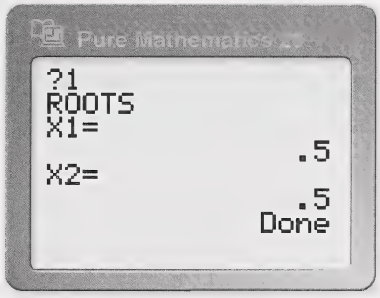
4. a.



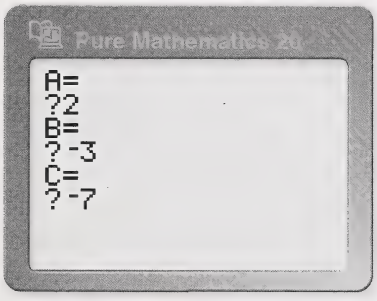


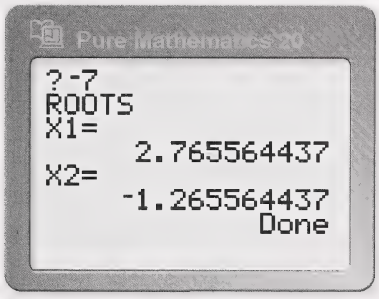
b.



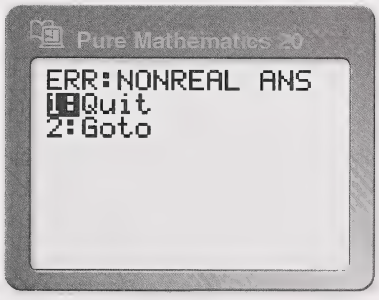


c.





5. When you try to solve $x^2 + 2x + 2 = 0$ using the quadratic formula program, an error message occurs.



The message appears because the program is designed to find real roots.

Section 3: Activity 3

1. a. Textbook question "Explore: Complete the Table," p. 187

Equation	Type of Solutions	Value of $b^2 - 4ac$
$x^2 + 2x + 1 = 0$	real and equal	0
$x^2 + 6x + 9 = 0$	real and equal	0
$x^2 + 4x + 3 = 0$	real and distinct	4
$x^2 + 6x + 7 = 0$	real and distinct	8
$x^2 + 5x + 7 = 0$	imaginary	-3
$x^2 + 6x + 10 = 0$	imaginary	-4

b. Textbook questions 1 to 3 of "Inquire," p. 187

1. a. The roots are real and equal because the discriminant is 0.
b. The roots are real and distinct because the discriminant is positive.
c. The roots are imaginary because the discriminant is negative.

2. a.
$$2.1 = -2d^2 + 4d$$
$$2d^2 - 4d + 2.1 = 0$$
$$\therefore a = 2,\ b = -4, \text{ and } c = 2.1$$
$$\therefore b^2 - 4ac = (-4)^2 - 4(2)(2.1)$$
$$= 16 - 16.8$$
$$= -0.8$$

Because $b^2 - 4ac < 0$, the equation has no real roots.

b. The jumper does not clear the bar because he or she did not, at any time, reach a height of 2.1 m.

3. If $b^2 - 4ac > 0$ and is not a perfect square, $\sqrt{b^2 - 4ac}$ will be irrational. The roots given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ will be irrational as well.

If $b^2 - 4ac > 0$ and is a perfect square, $\sqrt{b^2 - 4ac}$ will be rational and the roots obtained from the quadratic formula will be rational.

Both conclusions are based on the assumption that the coefficients a, b, and c are rational coefficients.

2. a. Textbook questions 1, 2, 3, 14, 15, 23, 26, 29, 30, 31, 35, 36, 42, and 43 of "Practice," pp. 189 and 190

1. $x^2 - 8x + 16 = 0$

$\therefore a = 1,\ b = -8,$ and $c = 16$

$$\begin{aligned} \therefore b^2 - 4ac &= (-8)^2 - 4(1)(16) \\ &= 64 - 64 \\ &= 0 \end{aligned}$$

Since $b^2 - 4ac = 0$, there are two equal real roots.

2. $x^2 - x - 5 = 0$

$\therefore a = 1,\ b = -1,$ and $c = -5$

$$\begin{aligned} \therefore b^2 - 4ac &= (-1)^2 - 4(1)(-5) \\ &= 1 + 20 \\ &= 21 \end{aligned}$$

Since $b^2 - 4ac > 0$, there are two distinct real roots.

3. $x^2 + 3x + 10 = 0$

$\therefore a = 1,\ b = 3,$ and $c = 10$

$$\begin{aligned} \therefore b^2 - 4ac &= 3^2 - 4(1)(10) \\ &= 9 - 40 \\ &= -31 \end{aligned}$$

Since $b^2 - 4ac < 0$, there are two imaginary roots.

Section 3: Activity 3 (continued)

14. $9n^2 = 5n$

$9n^2 - 5n = 0$

$\therefore\ a = 9,\ b = -5,\ c = 0$

$\therefore\ b^2 - 4ac = (-5)^2 - 4(9)(0)$

$= 25 - 0$

$= 25$

Since $b^2 - 4ac > 0$, there are two distinct real roots.

15. $9 = -4x^2$

$4x^2 + 9 = 0$

$\therefore\ a = 4,\ b = 0,$ and $c = 9$

$\therefore\ b^2 - 4ac = 0^2 - 4(4)(9)$

$= 0 - 144$

$= -144$

Since $b^2 - 4ac < 0$, there are two imaginary roots.

23. $\frac{x-1}{2} - x^2 = 3$

$x - 1 - 2x^2 = 6$

$-2x^2 + x - 7 = 0$

$2x^2 - x + 7 = 0$

$\therefore\ a = 2,\ b = -1,$ and $c = 7$

$\therefore\ b^2 - 4ac = (-1)^2 - 4(2)(7)$

$= 1 - 56$

$= -55$

Since $b^2 - 4ac < 0$, there are two imaginary roots.

26. $\sqrt{5}x^2 + 7x + 2\sqrt{5} = 0$

$\therefore\ a = \sqrt{5},\ b = 7,$ and $c = 2\sqrt{5}$

$$\begin{aligned}\therefore\ b^2 - 4ac &= 7^2 - 4\left(\sqrt{5}\right)\left(2\sqrt{5}\right)\\ &= 49 - 8(5)\\ &= 49 - 40\\ &= 9\end{aligned}$$

Since $b^2 - 4ac > 0$, there are two distinct real roots.

29. $f(x) = 6x^2 - 7x + 2$

The function intersects the x-axis when $f(x) = 0$.

$\therefore\ 6x^2 - 7x + 2 = 0$

$\therefore\ a = 6,\ b = -7,$ and $c = 2$

$$\begin{aligned}\therefore\ b^2 - 4ac &= (-7)^2 - 4(6)(2)\\ &= 49 - 48\\ &= 1\end{aligned}$$

Since $b^2 - 4ac > 0$, the equation has two distinct real roots. Therefore, $f(x) = 6x^2 - 7x + 2$ intersects the x-axis at two distinct points.

30. $y = 16x^2 + 24x + 9$

The function intersects the x-axis when $y = 0$.

$\therefore\ 16x^2 + 24x + 9 = 0$

$\therefore\ a = 16,\ b = 24,$ and $c = 9$

$$\begin{aligned}\therefore\ b^2 - 4ac &= 24^2 - 4(16)(9)\\ &= 576 - 576\\ &= 0\end{aligned}$$

Since $b^2 - 4ac = 0$, the equation has two equal real roots. Therefore, the function intersects the x-axis at exactly one point.

Section 3: Activity 3 (continued)

31. $f(x) = -x^2 + 5x - 8$

The function intersects the x-axis when $f(x) = 0$.

$\therefore\ -x^2 + 5x - 8 = 0$

$\therefore\ a = -1,\ b = 5,$ and $c = -8$

$$\begin{aligned}\therefore\ b^2 - 4ac &= 5^2 - 4(-1)(-8)\\ &= 25 - 32\\ &= -7\end{aligned}$$

Since $b^2 - 4ac < 0$, the equation has imaginary roots. Therefore, the function does not intersect the x-axis.

35. $x^2 + 4x - 2k = 0$

$\therefore\ a = 1,\ b = 4,$ and $c = -2k$

$$\begin{aligned}\therefore\ b^2 - 4ac &= 4^2 - 4(1)(-2k)\\ &= 16 + 8k\end{aligned}$$

For two imaginary roots,

$$\begin{aligned}b^2 - 4ac &< 0\\ 16 + 8k &< 0\\ 8k &< -16\\ k &< -2\end{aligned}$$

There are two imaginary roots when $k < -2$.

36. $2kx^2 - 4x + 3 = 0$

$\therefore\ a = 2k,\ b = -4,$ and $c = 3$

If the equation is quadratic, then $k \neq 0$.

$$\begin{aligned}\therefore\ b^2 - 4ac &= (-4)^2 - 4(2k)(3) \\ &= 16 - 24k\end{aligned}$$

For two distinct real roots,

$$\begin{aligned} b^2 - 4ac &> 0 \\ 16 - 24k &> 0 \\ 16 &> 24k \\ 24k &< 16 \\ k &< \frac{16}{24} \\ &< \frac{2}{3}\end{aligned}$$

There are two distinct real roots when $k < \frac{2}{3}$ and $k \neq 0$.

42. $y^2 + (k+2)y + 2k = 0$

$\therefore\ a = 1,\ b = k + 2,$ and $c = 2k$

$$\begin{aligned}\therefore\ b^2 - 4ac &= (k+2)^2 - 4(1)(2k) \\ &= k^2 + 4k + 4 - 8k \\ &= k^2 - 4k + 4\end{aligned}$$

For two equal real roots,

$$\begin{aligned} b^2 - 4ac &= 0 \\ k^2 - 4k + 4 &= 0 \\ (k-2)^2 &= 0\end{aligned}$$

$$\begin{aligned}\therefore\ k - 2 &= 0 \\ k &= 2\end{aligned}$$

There are two equal real roots when $k = 2$.

Section 3: Activity 3 (continued)

43. $mx^2 + 4x - 3 = 0$

$\therefore\ a = m,\ b = 4,$ and $c = -3$

$\therefore\ b^2 - 4ac = 4^2 - 4(m)(-3)$

$= 16 + 12m$

If the equation is quadratic, then $m \neq 0$.

a. For two real and distinct roots,

$$b^2 - 4ac > 0$$
$$16 + 12m > 0$$
$$12m > -16$$
$$m > \frac{-16}{12}$$
$$> -\frac{4}{3}$$

There are two real and distinct roots when $m > -\frac{4}{3}$ and $m \neq 0$.

b. For two real and equal roots,

$$b^2 - 4ac = 0$$
$$16 + 12m = 0$$
$$12m = -16$$
$$m = -\frac{4}{3}$$

There are two real and equal roots when $m = -\frac{4}{3}$.

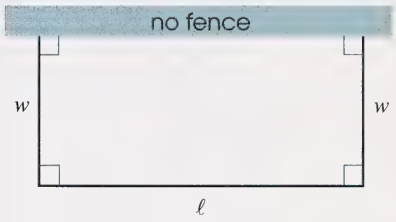
c. For two imaginary roots,

$$b^2 - 4ac < 0$$
$$16 + 12m < 0$$
$$12m < -16$$
$$m < -\frac{4}{3}$$

There are two imaginary roots when $m < -\frac{4}{3}$.

b. Textbook questions 45, 47, 49, 50, and 54.a. of "Applications and Problem Solving," p. 190

45. a. Let ℓ be the length (in metres) and let w be the width (in metres).



$$\therefore \ \ell + 2w = 30$$
$$\ell = 30 - 2w$$

$$\text{Area} = \ell w$$
$$100 = (30 - 2w)w$$
$$100 = 30w - 2w^2$$
$$2w^2 - 30w + 100 = 0$$
$$w^2 - 15w + 50 = 0$$

$$\therefore \ a = 1, \ b = -15, \text{ and } c = 50$$

$$\therefore \ b^2 - 4ac = (-15)^2 - 4(1)(50)$$
$$= 225 - 200$$
$$= 25$$

Because $b^2 - 4ac > 0$, solutions exist.

Use the quadratic formula.

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-15) \pm \sqrt{25}}{2(1)}$$
$$= \frac{15 \pm 5}{2}$$

$$\therefore \ w = \frac{15+5}{2} \quad \text{or} \quad w = \frac{15-5}{2}$$
$$= 10 \qquad\qquad = 5$$

If $w = 10$, then

$$\ell = 30 - 2w$$
$$= 30 - 2(10)$$
$$= 10$$

If $w = 5$, then

$$\ell = 30 - 2w$$
$$= 30 - 2(5)$$
$$= 20$$

The dimensions of the rectangle could be $10\,\text{m} \times 10\,\text{m}$ or $5\,\text{m} \times 20\,\text{m}$.

Section 3: Activity 3 (continued)

b. Let ℓ be the length (in metres), and let w be the width (in metres).

$$\therefore\ 2w+\ell = 25$$
$$\ell = 25-2w$$

$$\text{Area} = \ell w$$
$$100 = (25-2w)w$$
$$100 = 25w-2w^2$$
$$2w^2-25w+100=0$$

$$\therefore\ a=2,\ b=-25,\ \text{and}\ c=100$$

$$\therefore\ b^2-4ac = (-25)^2-4(2)(100)$$
$$= 625-800$$
$$= -175$$

Because $b^2-4ac<0$, no real roots exist. It is not possible to construct a rectangular enclosure 100 m^2 in area, if only 25 m of fencing is available.

47. If $h(t)=-5t^2+20t+80$ and $h(t)=100$, then

$$100 = -5t^2+20t+80$$
$$5t^2-20t+20=0$$
$$t^2-4t+4=0$$

$$\therefore\ a=1,\ b=-4,\ \text{and}\ c=4$$

$$\therefore\ b^2-4ac = (-4)^2-4(1)(4)$$
$$= 16-16$$
$$= 0$$

Because $b^2-4ac=0$, a real solution to the equation exists.

Yes, the object will reach a height of 100 m.

To determine when the object reaches 100 m, solve $t^2 - 4t + 4 = 0$ by factoring.

$$\begin{aligned} t^2 - 4t + 4 &= 0 \\ (t-2)^2 &= 0 \\ t - 2 &= 0 \\ t &= 2 \end{aligned}$$

The object reaches 100 m at $t = 2$ s.

49. If $ac < 0$, then

$$\begin{aligned} -ac &> 0 \\ 4(-ac) &> 4(0) && \leftarrow \text{Multiply both sides by 4.} \\ -4ac &> 0 \\ b^2 - 4ac &> b^2 + 0 && \leftarrow \text{Add } b^2 \text{ to both sides.} \\ b^2 - 4ac &> b^2 \end{aligned}$$

$\therefore\ b^2 - 4ac > 0$ ← This is true because $b^2 - 4ac > b^2$ and $b^2 \geq 0$.

Because $b^2 - 4ac > 0$, there are two distinct real roots.

50. $2x^2 + 5x + k = 0$

$\therefore\ a = 2,\ b = 5,$ and $c = k$

$$\begin{aligned} \therefore\ b^2 - 4ac &= 5^2 - 4(2)(k) \\ &= 25 - 8k \end{aligned}$$

If the equation has imaginary roots,

$$\begin{aligned} b^2 - 4ac &< 0 \\ 25 - 8k &< 0 \\ 25 &< 8k \\ 8k &> 25 \\ k &> \frac{25}{8} \\ k &> 3\frac{1}{8} \end{aligned}$$

Section 3: Activity 3 (continued)

Now, because k is the result of rolling a die, k must be 4, 5, or 6 in order for the roots of the equation to be imaginary.

$$\therefore P(4 \text{ or } 5 \text{ or } 6) = \frac{3}{6} \quad \begin{array}{l} \longleftarrow 3 \text{ successful outcomes} \\ \longleftarrow 6 \text{ possible outcomes} \end{array}$$

$$= \frac{1}{2}$$

The probability that the equation would have imaginary roots is $\frac{1}{2}$.

54. a. If the graphs of $y = 4x^2 + 6x + 7$ and $y = -2x + 4$ intersect, then $y = y$ at the points of intersection.

$$\therefore 4x^2 + 6x + 7 = -2x + 4$$

$$4x^2 + 8x + 3 = 0$$

$$\therefore a = 4,\ b = 8, \text{ and } c = 3$$

$$\therefore b^2 - 4ac = 8^2 - 4(4)(3)$$

$$= 64 - 48$$

$$= 16$$

Because $b^2 - 4ac > 0$, the equation has distinct and real roots; thus, the graphs of the functions intersect.

To determine the x-values at these points of intersection, solve the equation $4x^2 + 8x + 3 = 0$ using the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-8 \pm \sqrt{16}}{2(4)}$$

$$= \frac{-8 \pm 4}{8}$$

$$= \frac{-2 \pm 1}{2}$$

$$\therefore\ x = \frac{-2+1}{2} \quad \text{or} \quad x = \frac{-2-1}{2}$$
$$= -\frac{1}{2} \qquad\qquad = -\frac{3}{2}$$

Substitute $x = -\frac{1}{2}$ and $x = -\frac{3}{2}$ into $y = -2x + 4$ to find the y-values.

For $x = -\frac{1}{2}$,

$$y = -2\left(-\frac{1}{2}\right) + 4$$
$$= 1 + 4$$
$$= 5$$

For $x = -\frac{3}{2}$,

$$y = -2\left(-\frac{3}{2}\right) + 4$$
$$= 3 + 4$$
$$= 7$$

The graphs intersect at $\left(-\frac{1}{2}, 5\right)$ and $\left(-\frac{3}{2}, 7\right)$.

3. a. Textbook questions 1 to 5 and 6.a. and 6.c. of Investigation 1, "Exploring the Relationships," p. 192

1.

	Equation	Roots	Sum of Roots	Product of Roots	Value of a	Value of b	Value of c
a.	$x^2 + 5x + 6 = 0$	$-3, -2$	-5	6	1	5	6
b.	$x^2 - 9x + 20 = 0$	4, 5	9	20	1	-9	20
c.	$x^2 + 4x - 12 = 0$	-6, 2	-4	-12	1	4	-12
d.	$2x^2 - 7x + 3 = 0$	$\frac{1}{2}$, 3	$\frac{7}{2}$	$\frac{3}{2}$	2	-7	3
e.	$2x^2 + 3x + 1 = 0$	$-\frac{1}{2}$, -1	$-\frac{3}{2}$	$\frac{1}{2}$	2	3	1
f.	$3x^2 + 7x - 6 = 0$	$\frac{2}{3}$, -3	$\frac{-7}{3}$	-2	3	7	-6
g.	$x^2 + 8x = 0$	0, -8	-8	0	1	8	0
h.	$x^2 - 16 = 0$	-4, 4	0	-16	1	0	-16

2. The sum of the roots is determined by coefficients a and b.

3. Sum $= \frac{-b}{a}$

Section 3: Activity 3 (continued)

4. The product of the roots is determined by coefficients a and c.

5. Product $= \frac{c}{a}$

6. a. $x^2 + 6x + 8 = 0$

$\therefore\ a = 1,\ b = 6,$ and $c = 8$

$$\text{Sum} = \frac{-b}{a} \qquad \text{Product} = \frac{c}{a}$$
$$= \frac{-6}{1} \qquad = \frac{8}{1}$$
$$= -6 \qquad = 8$$

c. $2x^2 - 3x - 5 = 0$

$\therefore\ a = 2,\ b = -3,$ and $c = -5$

$$\text{Sum} = \frac{-b}{a} \qquad \text{Product} = \frac{c}{a}$$
$$= \frac{-(-3)}{2} \qquad = \frac{-5}{2}$$
$$= \frac{3}{2} \qquad = -\frac{5}{2}$$

b. Textbook questions 1 to 3 of Investigation 2, "Using Algebra," p. 192

1. a. $r_1 + r_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{-2b}{2a}$$
$$= \frac{-b}{a}$$

b. The result is the same: sum $= \frac{-b}{a}$.

2. a. $r_1 \times r_2 = \left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)$

$$= \frac{b^2-\left(\sqrt{b^2-4ac}\right)^2}{(2a)^2}$$

$$= \frac{b^2-\left(b^2-4ac\right)}{4a^2}$$

$$= \frac{b^2-b^2+4ac}{4a^2}$$

$$= \frac{4ac}{4a^2}$$

$$= \frac{c}{a}$$

b. The result is the same: product $= \frac{c}{a}$.

3. $ax^2+bx+c=0$

$x^2+\frac{b}{a}x+\frac{c}{a}=0$ ← Divide by a.

$x^2-\left(\frac{-b}{a}\right)x+\frac{c}{a}=0$ ← $-\left(\frac{-b}{a}\right)=\frac{b}{a}$

$\therefore x^2-(\text{sum of roots})x+(\text{product of roots})=0$, because sum of roots $=\frac{-b}{a}$ and product of roots $=\frac{c}{a}$.

c. Textbook questions 1.a. to 1.c. and 2.a. to 2.c. of Investigation 3, "Working Backward," p. 193

1. a. $x^2-(\text{sum of roots})x+(\text{product of roots})=0$

$$x^2-2x+3=0$$

b. $x^2-(\text{sum of roots})x+(\text{product of roots})=0$

$$x^2-(-1)x+5=0$$

$$x^2+x+5=0$$

Section 3: Activity 3 (continued)

c. $x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$

$$x^2 - (-2)x + (-2) = 0$$

$$x^2 + 2x - 2 = 0$$

2. a. Sum of roots $= 3 + 5$
$= 8$

Product of roots $= 3(5)$
$= 15$

$\therefore\ x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$

$$x^2 - 8x + 15 = 0$$

An equation with roots 3 and 5 is $x^2 - 8x + 15 = 0$.

b. Sum of roots $= 4 + (-1)$
$= 3$

Product of roots $= 4(-1)$
$= -4$

$\therefore\ x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$

$$x^2 - 3x + (-4) = 0$$

$$x^2 - 3x - 4 = 0$$

An equation with roots 4 and -1 is $x^2 - 3x - 4 = 0$.

c. Sum of roots $= \frac{2}{3} + (-3)$
$= \frac{2}{3} - \frac{9}{3}$
$= -\frac{7}{3}$

Product of roots $= \frac{2}{3}(-3)$
$= -2$

$\therefore\ x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$

$$x^2 - \left(-\frac{7}{3}\right)x + (-2) = 0$$

$$x^2 + \frac{7}{3}x - 2 = 0$$

$$3x^2 + 7x - 6 = 0$$

An equation with roots $\frac{2}{3}$ and -3 is $3x^2 + 7x - 6 = 0$.

Section 3: Follow-up Activities

Extra Help

1. a. $2x^2 - 2x - 3 = 0$

$\therefore\ a = 2,\ b = -2,$ and $c = -3$

$$\begin{aligned}\therefore\ b^2 - 4ac &= (-2)^2 - 4(2)(-3)\\ &= 4 + 24\\ &= 28\end{aligned}$$

Since $b^2 - 4ac > 0$, the equation has two real and distinct roots.

b. Using the quadratic formula,

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\\ &= \frac{-(-2) \pm \sqrt{28}}{2(2)} \quad \leftarrow b^2 - 4ac = 28\\ &= \frac{2 \pm 2\sqrt{7}}{4}\\ &= \frac{\overset{1}{\cancel{2}}\left(1 \pm \sqrt{7}\right)}{\underset{2}{\cancel{4}}}\\ &= \frac{1 \pm \sqrt{7}}{2}\end{aligned}$$

The roots are $\frac{1+\sqrt{7}}{2}$ and $\frac{1-\sqrt{7}}{2}$.

2. a. $2x^2 - 2x + 7 = 0$

$\therefore\ a = 2,\ b = -2,$ and $c = 7$

$$\begin{aligned}\therefore\ b^2 - 4ac &= (-2)^2 - 4(2)(7)\\ &= 4 - 56\\ &= -52\end{aligned}$$

Since $b^2 - 4ac < 0$, the equation has two imaginary roots.

b. Using the quadratic formula,

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\\ &= \frac{-(-2) \pm \sqrt{-52}}{2(2)} \quad \leftarrow b^2 - 4ac = -52\\ &= \frac{2 \pm 2i\sqrt{13}}{4}\\ x &= \frac{\overset{1}{\cancel{2}}\left(1 \pm i\sqrt{13}\right)}{\underset{2}{\cancel{4}}}\\ x &= \frac{1 \pm i\sqrt{13}}{2}\end{aligned}$$

The roots are $\frac{1+i\sqrt{13}}{2}$ and $\frac{1-i\sqrt{13}}{2}$.

Extra Help (continued)

3. a. $x^2 - 6x + 9 = 0$

$\therefore\ a = 1,\ b = -6,$ and $c = 9$

$$\therefore\ b^2 - 4ac = (-6)^2 - 4(1)(9)$$
$$= 36 - 36$$
$$= 0$$

Since $b^2 - 4ac = 0$, there are two real and equal roots.

b. Using the quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-6) \pm \sqrt{0}}{2(1)} \quad \longleftarrow b^2 - 4ac = 0$$
$$= \frac{6}{2}$$
$$= 3$$

The roots are 3 and 3.

Enrichment

1. Textbook questions 1 to 4 of Investigation 1, "Investigating ϕ," p. 195

1. a. $\ell^2 - \ell - 1 = 0$

$\therefore\ a = 1,\ b = -1,$ and $c = -1$

Using the quadratic formula,

$$\ell = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$
$$= \frac{1 \pm \sqrt{1+4}}{2}$$
$$= \frac{1 \pm \sqrt{5}}{2}$$

Because ϕ is the positive root, $\phi = \frac{1+\sqrt{5}}{2}$.

b. $\phi \doteq 1.618$

c. Answers may vary. The large rectangle near the centre of the painting along with the slim rectangle on its right forms a golden rectangle.

2. a. $\phi^2 = \left(\frac{1+\sqrt{5}}{2}\right)^2$

$= \frac{1+2\sqrt{5}+5}{4}$

$= \frac{6+2\sqrt{5}}{4}$

$= \frac{3+\sqrt{5}}{2}$

b. $\phi^2 \doteq 2.618$

c. $\phi^2 = \phi + 1$

d. $\phi^2 - \phi = \frac{3+\sqrt{5}}{2} - \frac{1+\sqrt{5}}{2}$

$= \frac{\left(3+\sqrt{5}\right)-\left(1+\sqrt{5}\right)}{2}$

$= \frac{3+\sqrt{5}-1-\sqrt{5}}{2}$

$= \frac{2}{2}$

$= 1$

This is the same result as in 2.c. because

$\phi^2 - \phi = 1$

$\phi^2 = \phi + 1$

Enrichment (continued)

3. a. $\frac{1}{\phi} = \frac{1}{\frac{1+\sqrt{5}}{2}}$

$= \frac{2}{1+\sqrt{5}}$

$= \frac{2}{1+\sqrt{5}} \times \frac{1-\sqrt{5}}{1-\sqrt{5}}$

$= \frac{2(1-\sqrt{5})}{1-5}$

$= \frac{\overset{1}{\cancel{2}}(1-\sqrt{5})}{\underset{-2}{\cancel{-4}}}$

$= \frac{1-\sqrt{5}}{-2}$

$= \frac{\sqrt{5}-1}{2}$

b. $\frac{1}{\phi} \doteq 0.618$

c. $\frac{1}{\phi} = \phi - 1$

d. $\phi - \frac{1}{\phi} = \frac{1+\sqrt{5}}{2} - \frac{\sqrt{5}-1}{2}$

$= \frac{(1+\sqrt{5}) - (\sqrt{5}-1)}{2}$

$= \frac{1+\sqrt{5}-\sqrt{5}+1}{2}$

$= \frac{2}{2}$

$= 1$

This is the same result as in 3.c. because

$\phi - \frac{1}{\phi} = 1$

$-\frac{1}{\phi} = -\phi + 1$

$\frac{1}{\phi} = \phi - 1$

4. a.

LS	RS
ϕ	$\dfrac{1}{\phi-1}$
$=\dfrac{1+\sqrt{5}}{2}$	$=\dfrac{1}{\frac{1+\sqrt{5}}{2}-1}$
	$=\dfrac{1}{\frac{1+\sqrt{5}-2}{2}}$
	$=\dfrac{2}{\sqrt{5}-1}\times\dfrac{\sqrt{5}+1}{\sqrt{5}+1}$
	$=\dfrac{2\left(\sqrt{5}+1\right)}{5-1}$
	$=\dfrac{\overset{1}{\cancel{2}}\left(1+\sqrt{5}\right)}{\underset{2}{\cancel{4}}}$
	$=\dfrac{1+\sqrt{5}}{2}$

LS = RS

b.

LS	RS
ϕ	$2-\dfrac{1}{\phi^2}$
$=\dfrac{1+\sqrt{5}}{2}$	$=2-\dfrac{1}{\left(\frac{1+\sqrt{5}}{2}\right)^2}$
	$=2-\dfrac{1}{\frac{1+2\sqrt{5}+5}{4}}$
	$=2-\dfrac{4}{6+2\sqrt{5}}$
	$=2-\dfrac{2}{3+\sqrt{5}}\times\dfrac{3-\sqrt{5}}{3-\sqrt{5}}$
	$=2-\dfrac{2\left(3-\sqrt{5}\right)}{9-5}$
	$=2-\dfrac{\overset{1}{\cancel{2}}\left(3-\sqrt{5}\right)}{\underset{2}{\cancel{4}}}$
	$=\dfrac{4-\left(3-\sqrt{5}\right)}{2}$
	$=\dfrac{4-3+\sqrt{5}}{2}$
	$=\dfrac{1+\sqrt{5}}{2}$

LS = RS

2. Textbook questions 1 and 2 of Investigation 2, "Geometry and ϕ," p. 195

1. $x^2=1^2+2^2$

$x^2=1+4$

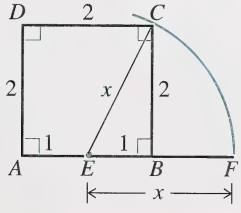
$x^2=5$

$x=\sqrt{5}$

$\therefore\ \dfrac{AF}{AB}=\dfrac{1+x}{2}$

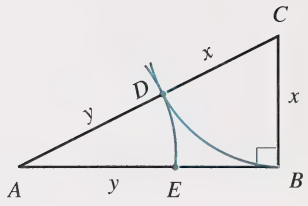
$=\dfrac{1+\sqrt{5}}{2}$

$=\phi$



Enrichment (continued)

2. Let $BC = x$, and let $AD = y$.



$\therefore\ AB = 2x$

Solve for y.

$$(AB)^2 + (BC)^2 = (AC)^2$$
$$(2x)^2 + x^2 = (x+y)^2$$
$$4x^2 + x^2 = x^2 + 2xy + y^2$$
$$5x^2 = x^2 + 2xy + y^2$$
$$0 = y^2 + 2xy - 4x^2$$

$\therefore\ a = 1,\ b = 2x,$ and $c = -4x^2$

$$\therefore y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-2x \pm \sqrt{(2x)^2 - 4(1)(-4x^2)}}{2(1)}$$
$$= \frac{-2x \pm \sqrt{4x^2 + 16x^2}}{2}$$
$$= \frac{-2x \pm \sqrt{20x^2}}{2}$$
$$= \frac{-2x \pm 2x\sqrt{5}}{2}$$
$$= \frac{2x(-1 \pm \sqrt{5})}{2}$$
$$= x(\pm\sqrt{5} - 1)$$

Because $y > 0$, $y = x\left(\sqrt{5}-1\right)$.

$$\therefore \frac{AB}{AE} = \frac{2x}{y}$$

$$= \frac{2x}{x\left(\sqrt{5}-1\right)}$$

$$= \frac{2}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1}$$

$$= \frac{\overset{1}{\cancel{2}}\left(\sqrt{5}+1\right)}{\underset{2}{\cancel{4}}}$$

$$= \frac{\sqrt{5}+1}{2}$$

$$= \phi$$

3. Textbook questions 1 to 3 of Investigation 3, "Fibonacci Numbers and ϕ," p. 196

1. The Fibonacci sequence is 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ….

Compare the ratio of consecutive terms to ϕ.

t_k	t_{k+1}	$\frac{t_{k+1}}{t_k}$
1	1	1
1	2	2
2	3	1.5
3	5	1.66…
5	8	1.6
8	13	1.625
13	21	1.615 38…
21	34	1.619 04…
34	55	1.617 64…
55	89	1.618 18…
89	144	1.617 97…
144	233	1.618 05…
233	377	1.618 02…
377	610	1.618 03…

The ratio $\frac{610}{377}$ correctly shows the first six digits of ϕ.

Enrichment (continued)

2. a. $1+\dfrac{1}{1+\frac{1}{1+1}}=1+\dfrac{1}{1+\frac{1}{2}}$

$=1+\dfrac{1}{\frac{3}{2}}$

$=1+\dfrac{2}{3}$

$=\dfrac{5}{3}$

b. $1+\dfrac{1}{1+\dfrac{1}{1+\frac{1}{1+1}}}=1+\dfrac{1}{1+\dfrac{1}{1+\frac{1}{1+\frac{1}{2}}}}$

$=1+\dfrac{1}{1+\dfrac{1}{1+\frac{1}{\frac{3}{2}}}}$

$=1+\dfrac{1}{1+\dfrac{1}{1+\frac{2}{3}}}$

$=1+\dfrac{1}{1+\frac{1}{\frac{5}{3}}}$

$=1+\dfrac{1}{1+\frac{3}{5}}$

$=1+\dfrac{1}{\frac{8}{5}}$

$=1+\dfrac{5}{8}$

$=\dfrac{13}{8}$

c. The continued fractions are equivalent to ratios of consecutive terms of the Fibonacci sequence.

d. $\dfrac{34}{21}=1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\frac{1}{1+1}}}}}}$

3. a. 2, 5, 7, 12, 19, 31, 50, 81, 131, 212, …

b.

t_k	t_{k+1}	$\frac{t_{k+1}}{t_k}$
2	5	2.5
5	7	1.4
7	12	1.714 28...
12	19	1.583 33...
19	31	1.631 57...
31	50	1.612 90...
50	81	1.62
81	131	1.617 28...
131	212	1.618 32...

The ratios of successive terms converge to ϕ.

c. Answers may vary. A sample answer is given.

3, 7, 10, 17, 27, 44, 71, 115, 186, 301, ...

t_k	t_{k+1}	$\frac{t_{k+1}}{t_k}$
3	7	2.333 33...
7	10	1.428 57...
10	17	1.7
17	27	1.588 23...
27	44	1.629 62...
44	71	1.613 63...
71	115	1.619 71...
115	186	1.617 39...
186	301	1.618 27...

The ratios of successive terms converge to ϕ.

4. Textbook question 1 of Investigation 4, "ϕ in Architecture, Design, and Nature," p. 196

1. $\frac{\text{height}}{\text{width}} \doteq \phi$

$\text{height} \doteq \phi \times \text{width}$

$\doteq (1.618\,033\,989)(29)$

$\doteq 47$

The height is approximately 47 m.

Pure Mathematics 20
Student Module Booklet
Module 3

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